SIGGRAPH2010

The People Behind the Pixels

Triangle Surfaces with Discrete Equivalence Classes

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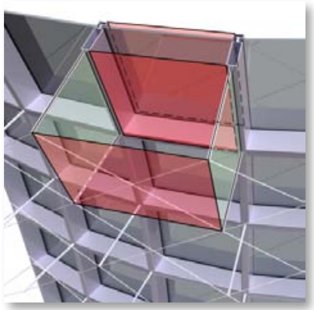
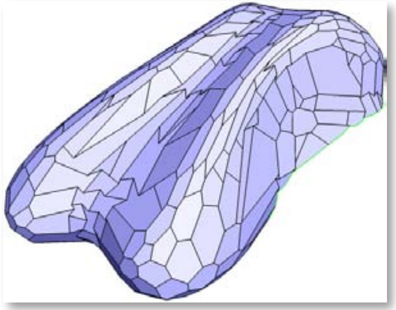
Scott Schaefer
TEXAS A&M UNIVERSITY

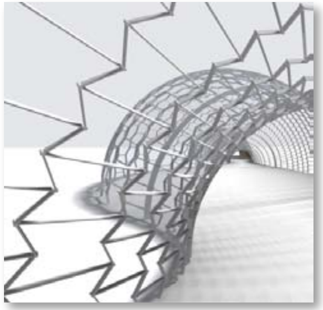
Introduction



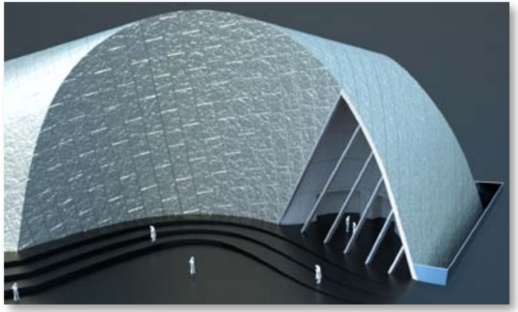
Liu et al. [2006]



Cutler and Whiting [2007]

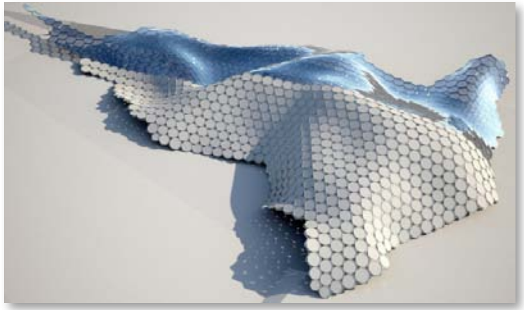


Pottmann et al. [2007]



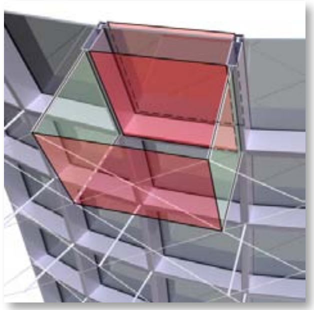
Killian et al. [2008]

Pottmann et al. [2008]



Schiftner et al. [2009]

Introduction



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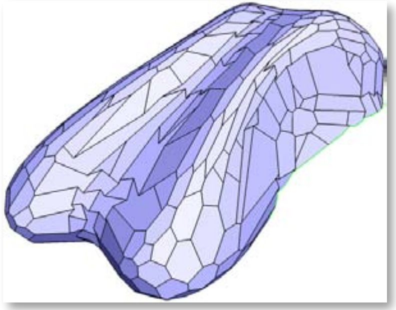

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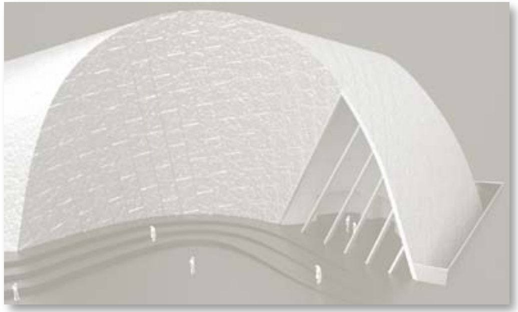

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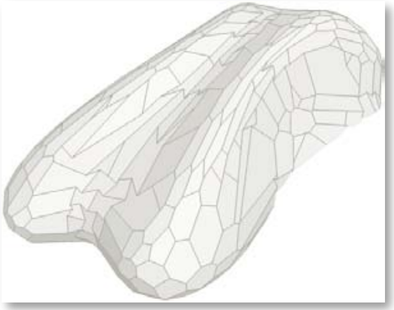

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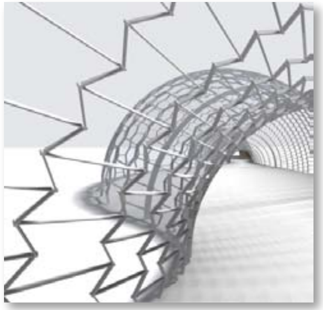

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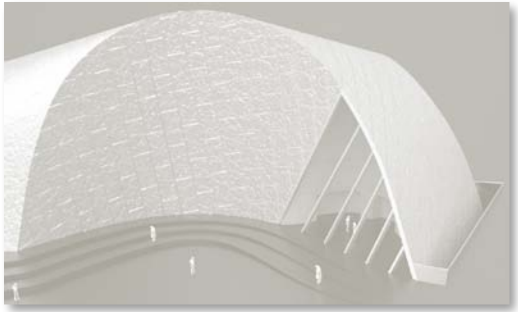

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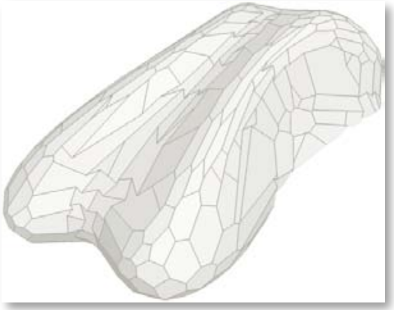

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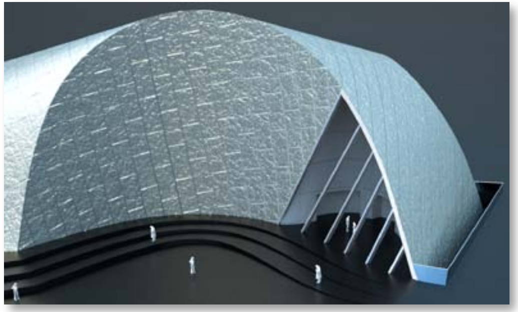

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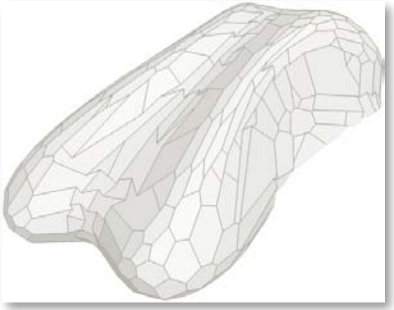

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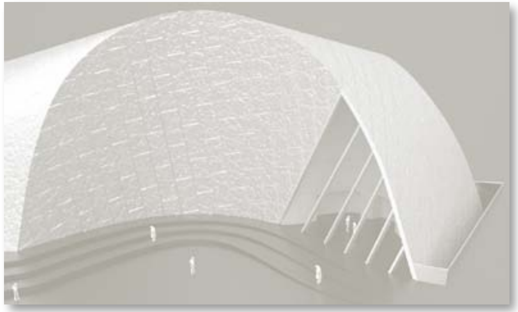

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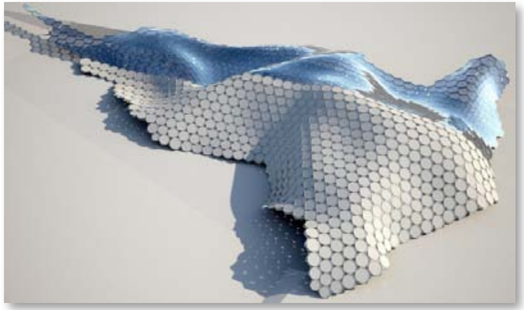
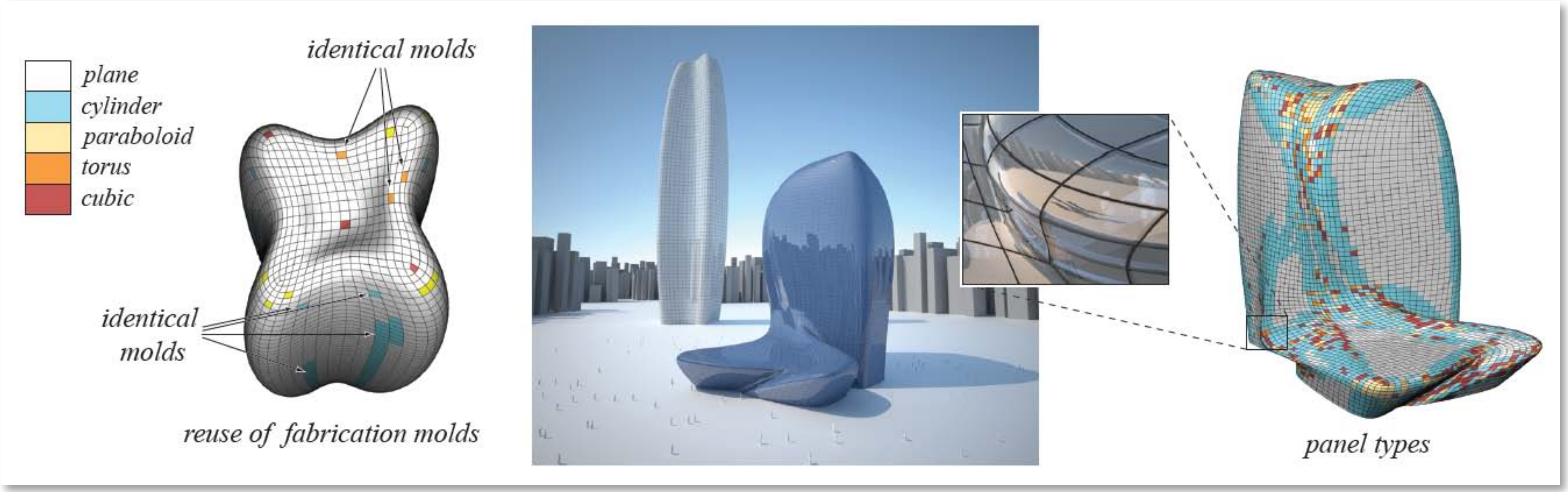


Schiftner et al. [2009]

Economy

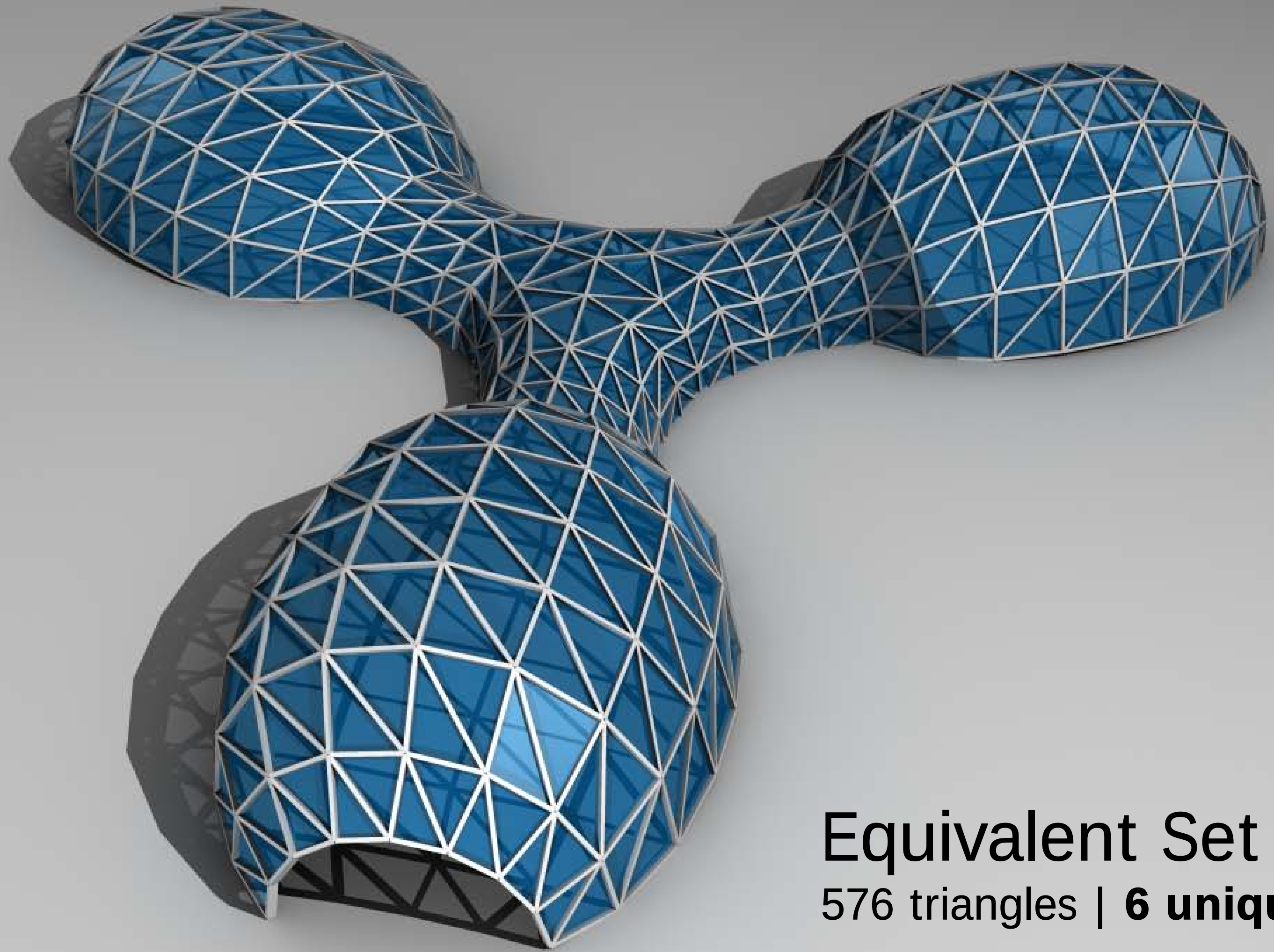
Paneling Architectural Freeform Surfaces

Michael Eigensatz, Martin Kilian, Alexander Schiftner, Niloy J. Mitra, Helmut Pottmann and Mark Pauly



Motivation

Beijing Aquatic Center

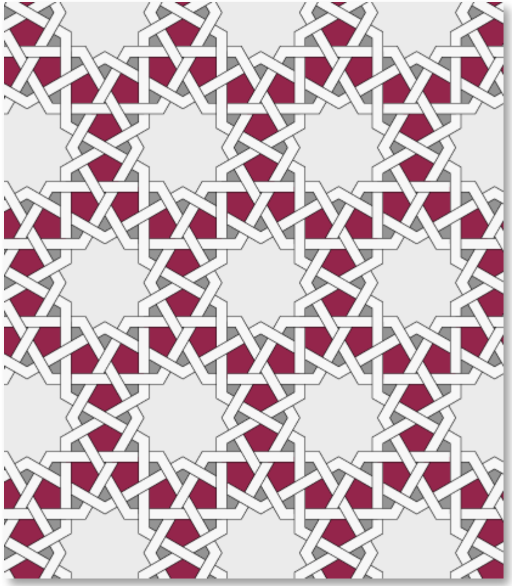
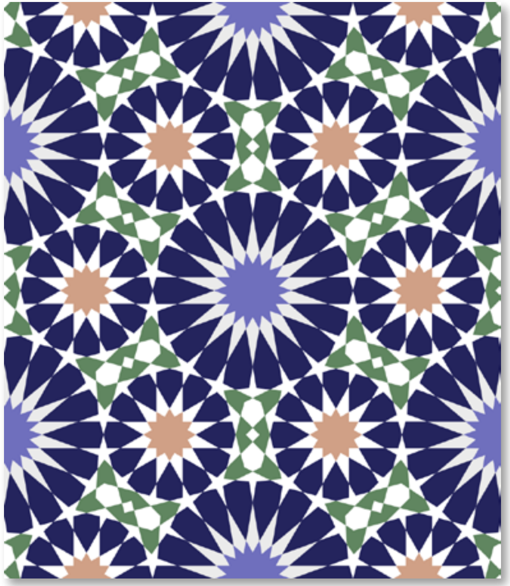
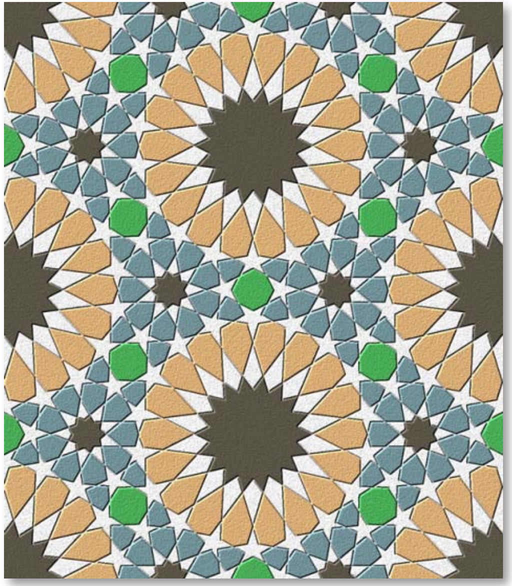


Equivalent Set Surface

576 triangles | **6 unique triangle**

Patterns – 2D

Planar patterns generated by Craig Kaplan [2004]

Patterns – 3D

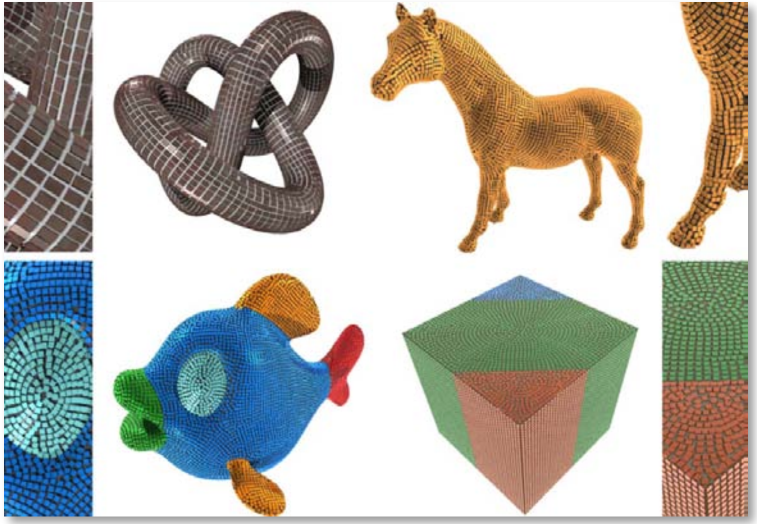
Quad parameterization of planar patterns [2009]

Mosaic – 2D

Kim & Pellacini [2002]

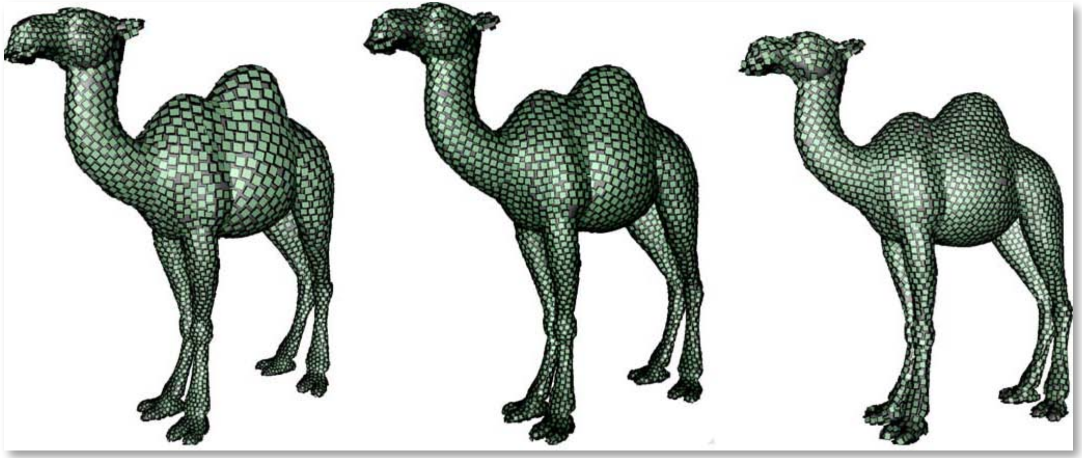
Elber & Wolberg [2003]

Mosaic – 3D

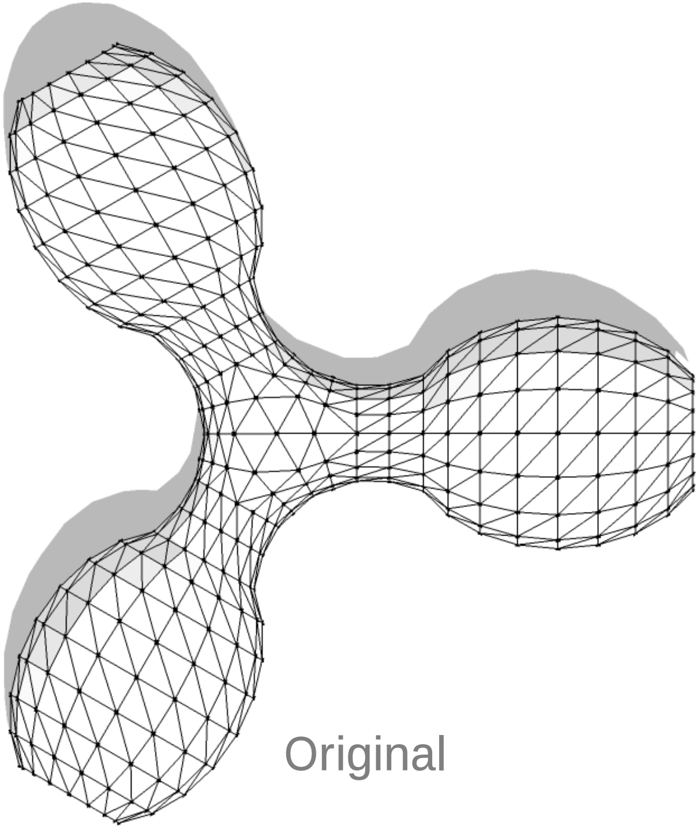


Lai et al. [2006]

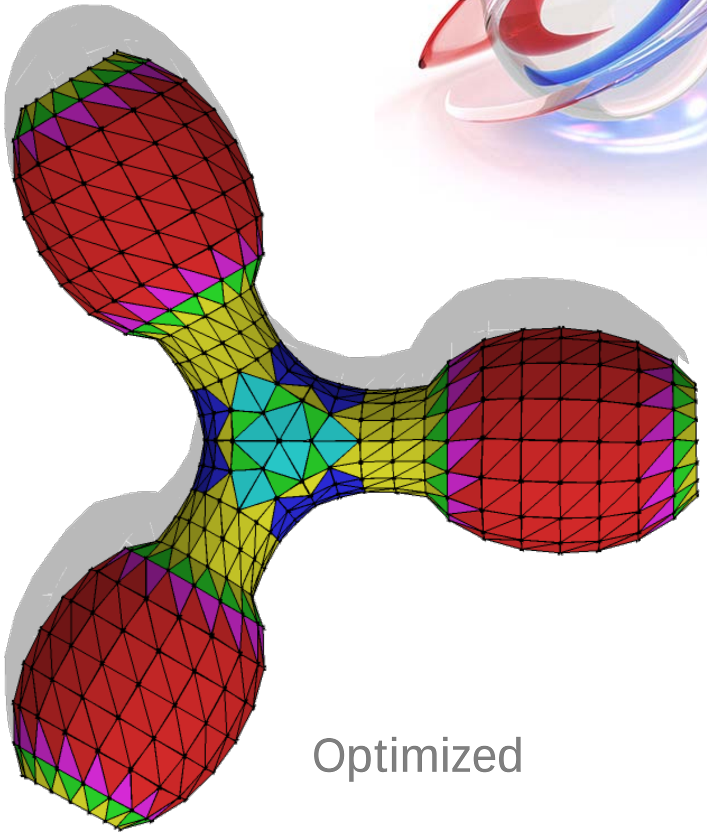
Passo & Walter [2008]



Equivalent Set Surface



Original



Optimized

Discrete Equivalence Classes

Input Shape

Clustering

Polygon Assignment &
Canonical Triangles

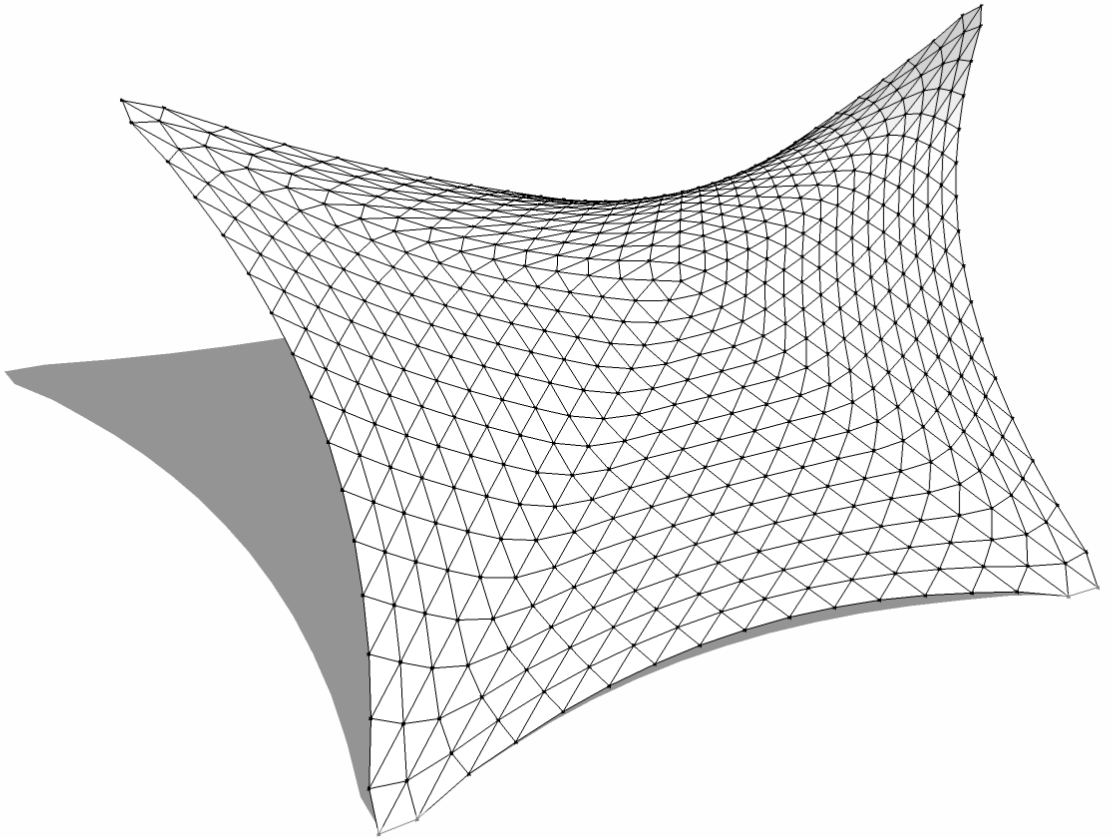
Modified
Geometry

Rigid Transformation

Mesh of Canonical
Triangles

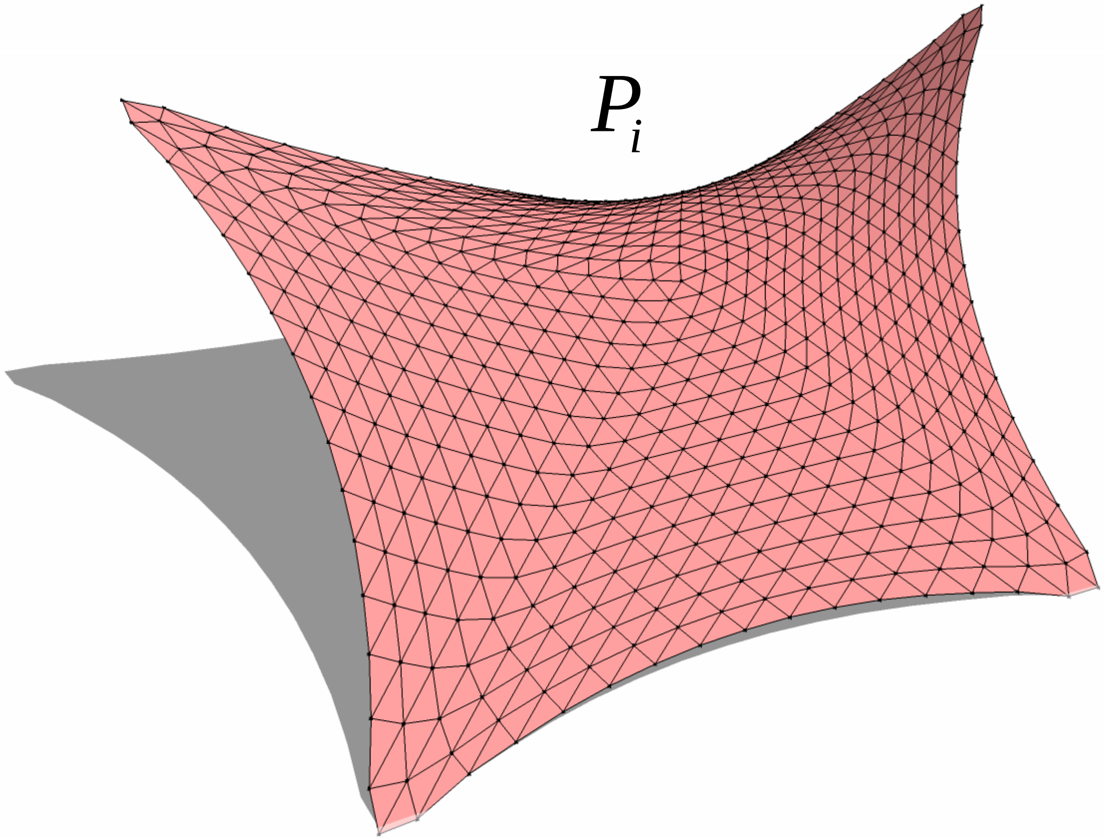
Global Linear Optimization

Example

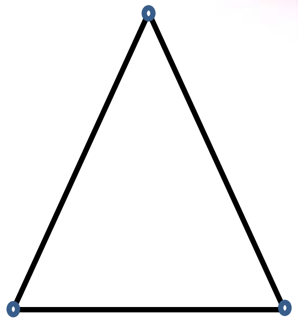


5-Point Tensile Roof
1280 triangles

Canonical Triangle



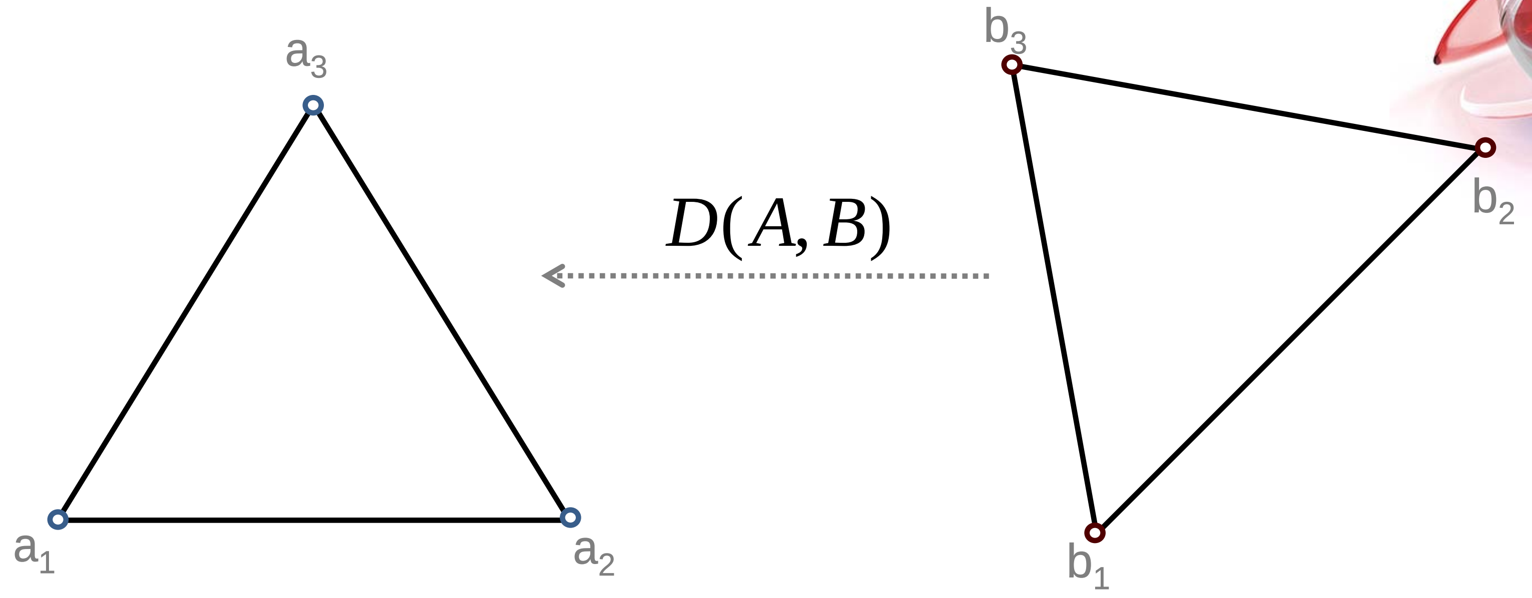
P_i



$C_{ind(i)}$

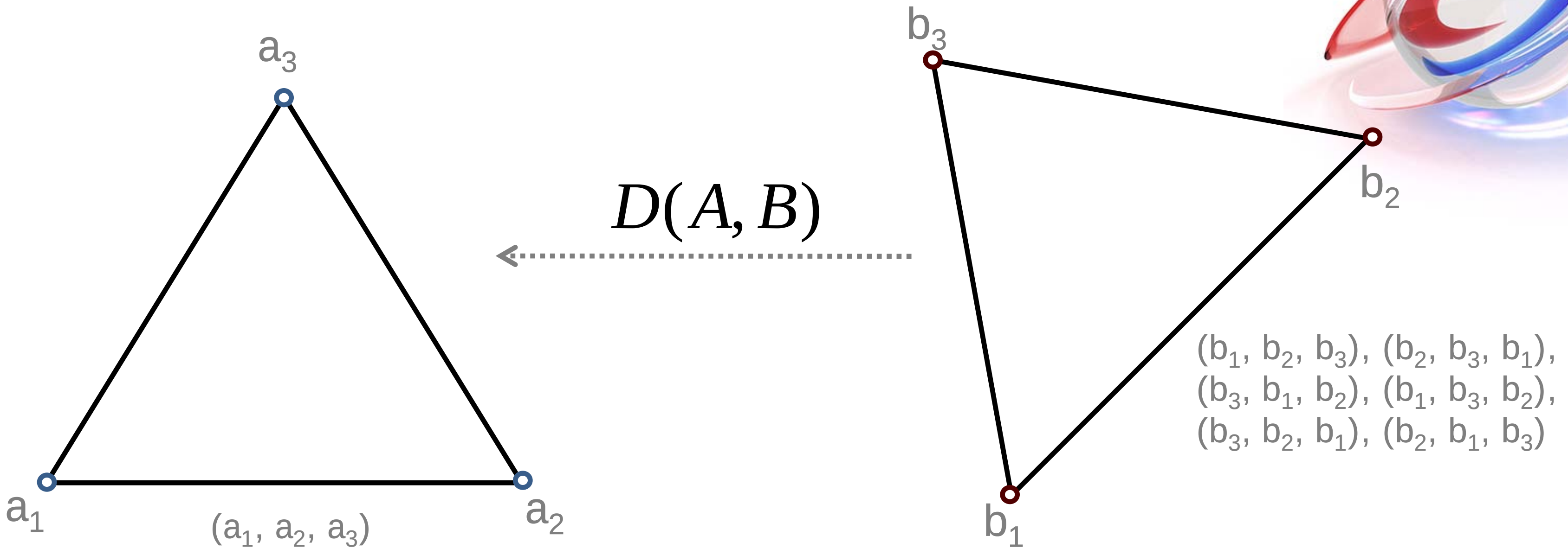
$$\min_{C_j, ind} \sum_i D(P_i, C_{ind(i)})$$

Triangle Similarity



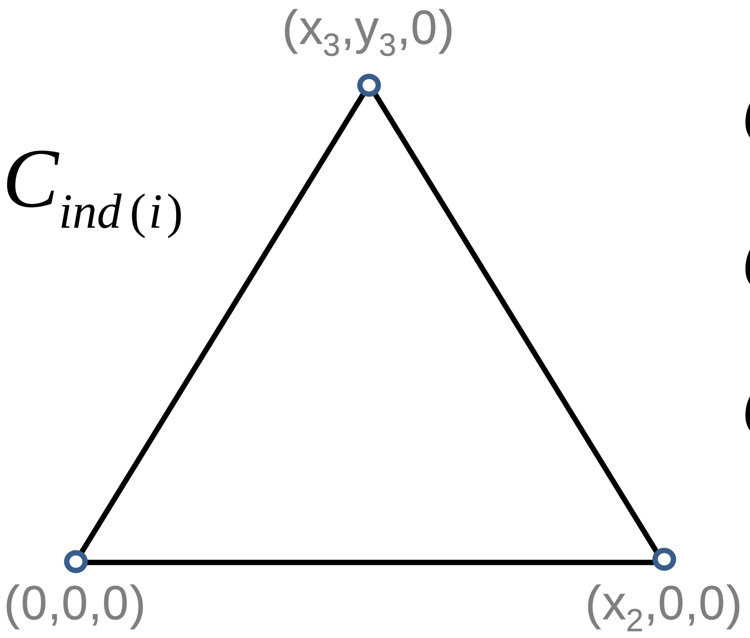
$$D(A,B) = \min_{R^T R = I, T, j} \sum_{l=1}^{3} | \underbrace{R b_{perm(j,l)} + T}_{\text{Transform B}} - a_l |^2$$

Triangle Similarity



$$D(A,B) = \min_{R^T R = I, T, j} \sum_{l=1}^{3} \left| \boxed{R} b_{\boxed{perm(j,l)}} + \boxed{T} - a_l \right|^2$$

Canonical Triangle



$$C_{j,1} = (0,0,0)$$

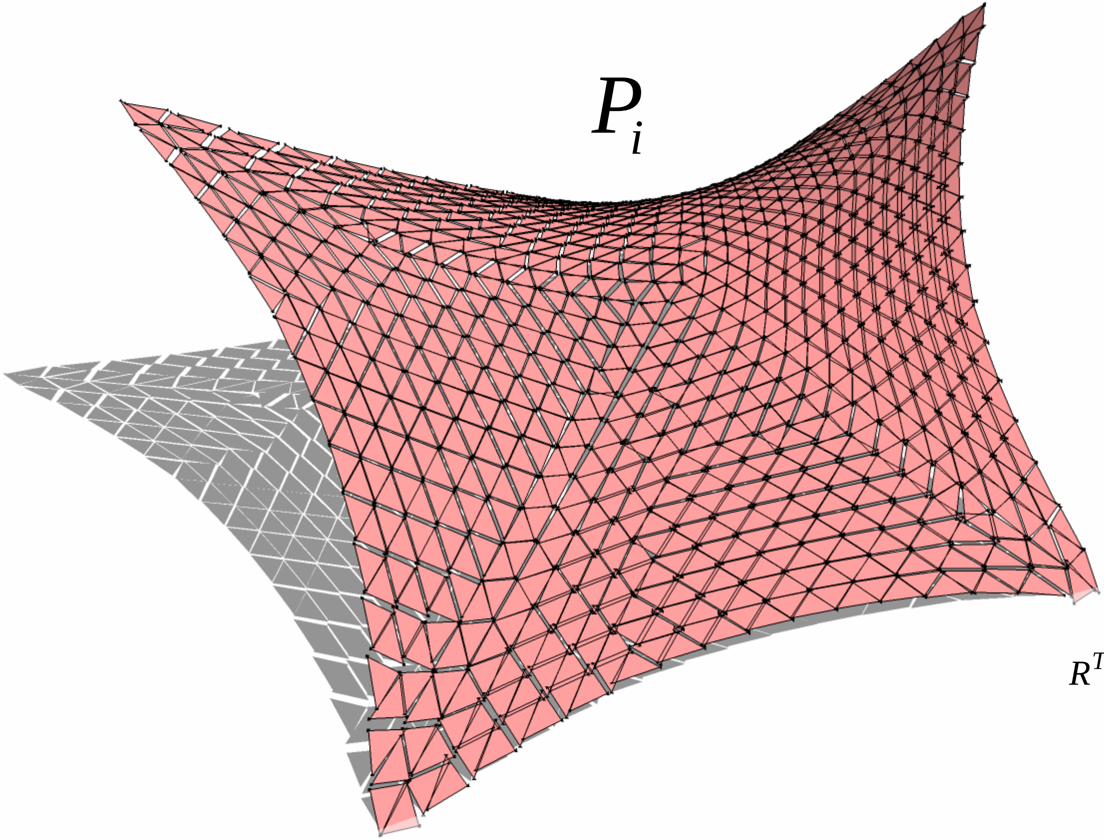
$$C_{j,2} = (x_2,0,0)$$

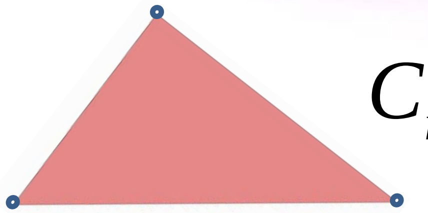
$$C_{j,3} = (x_3, y_3,0)$$

Nonlinear Minimization

$$\min_{C_j, ind} \sum_i D(P_i, C_{ind(i)})$$

Canonical Triangle



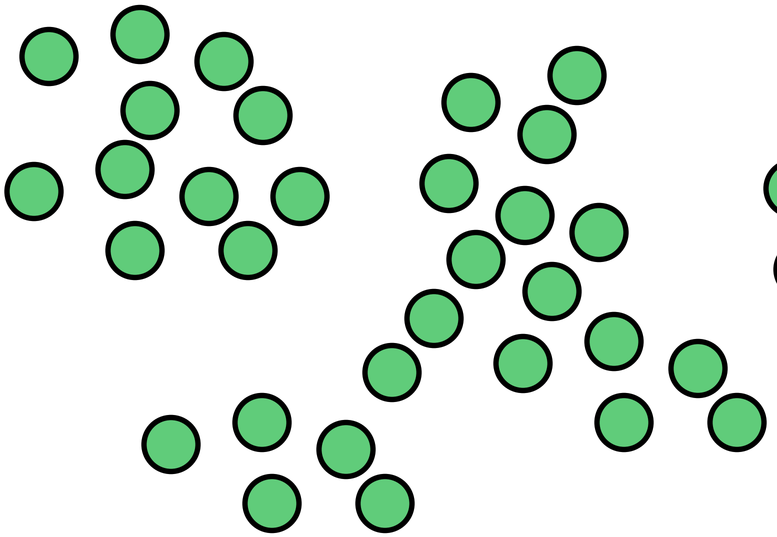


$C_{ind(i)}$

$$\min_{R^T R=I,T,j} \sum_{l=1}^{3} | R\boxed{C}_{perm(j,l)} + T - \boxed{P_l} |^2$$

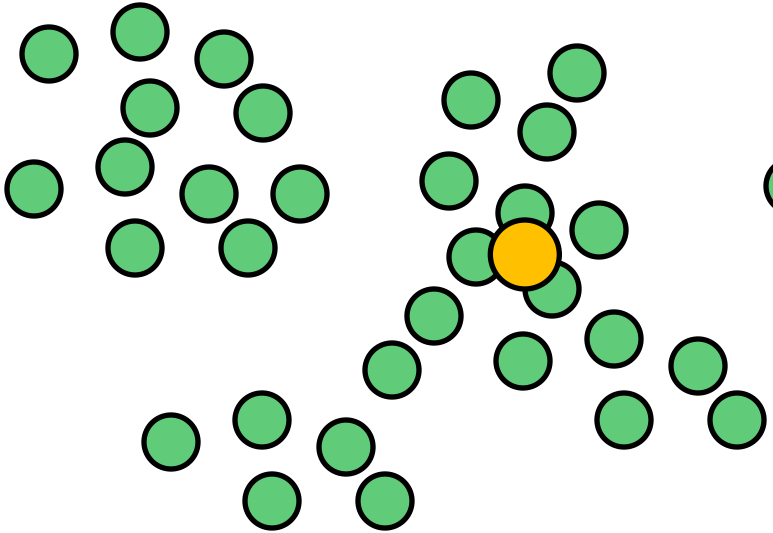
Rigid Transformation

Adaptive K-Means Clustering



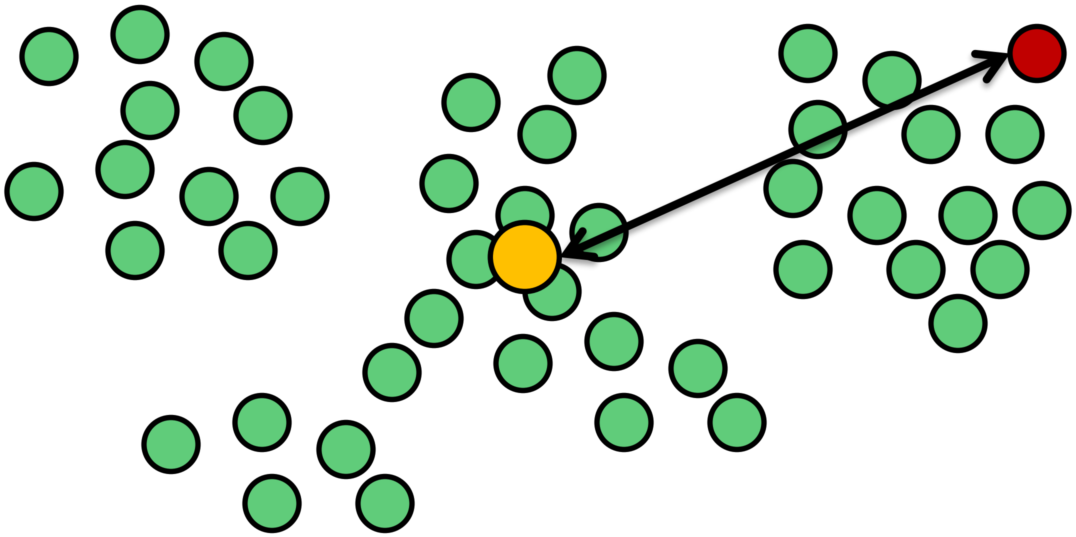
Each triangle is represented as a point

Adaptive K-Means Clustering



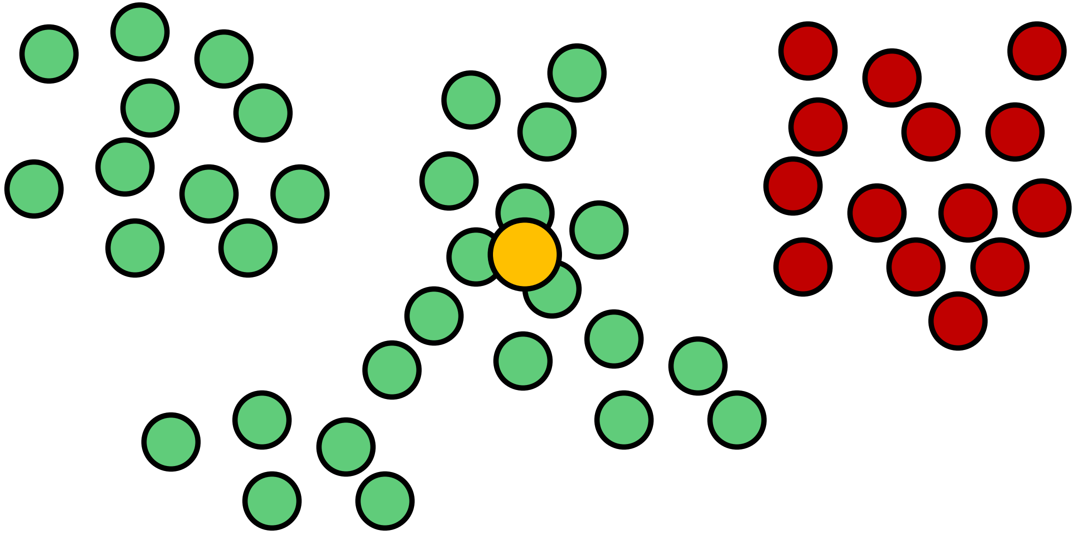
Compute center of the cluster
using nonlinear search

Adaptive K-Means Clustering



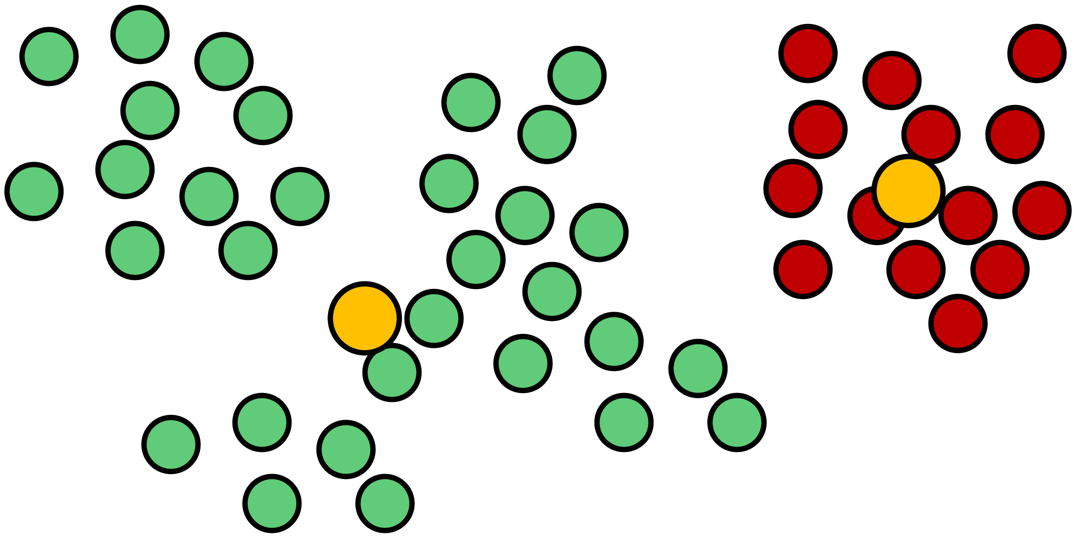
Assign the farthest point to a new cluster

Adaptive K-Means Clustering



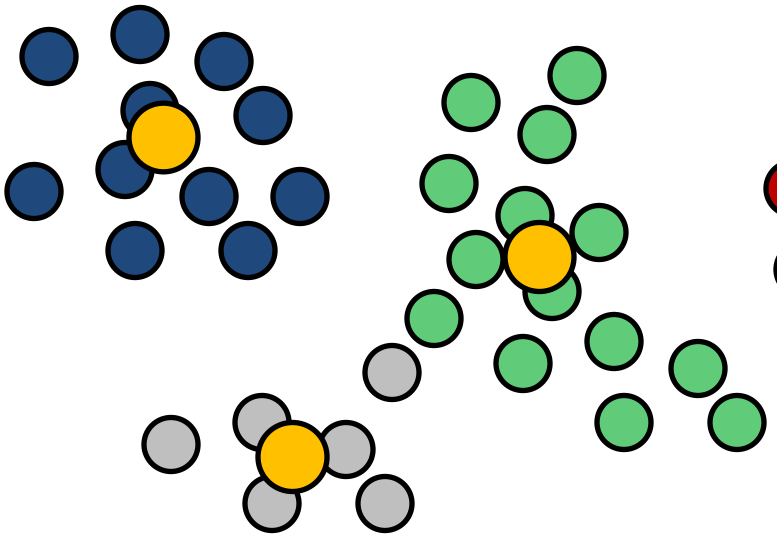
Reassign points to available clusters

Adaptive K-Means Clustering



Process continues to generate more clusters

Adaptive K-Means Clustering



Process continues to generate more clusters

Clustering

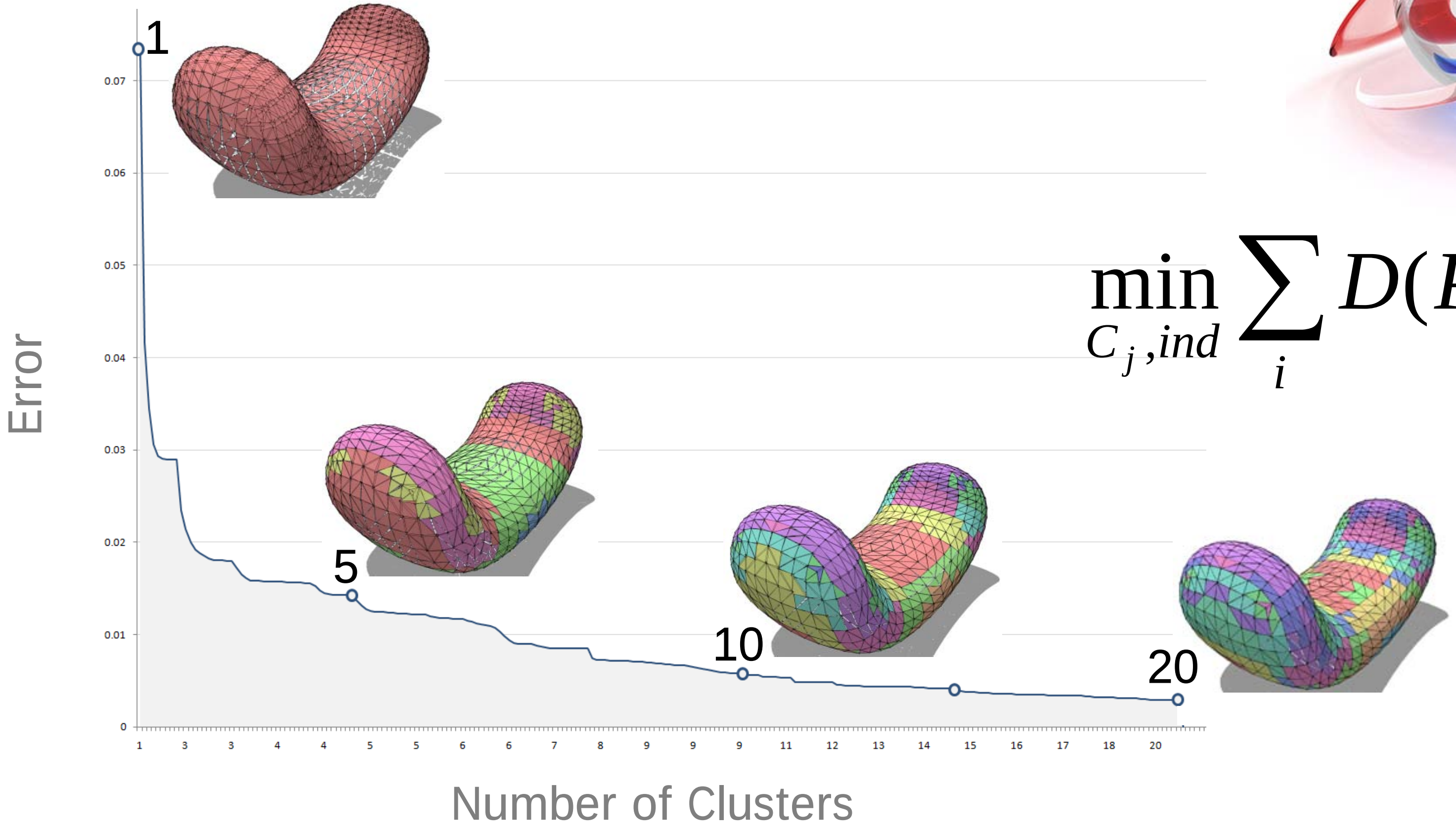
Polygon Assignment

Canonical Polygons

Generate Clusters

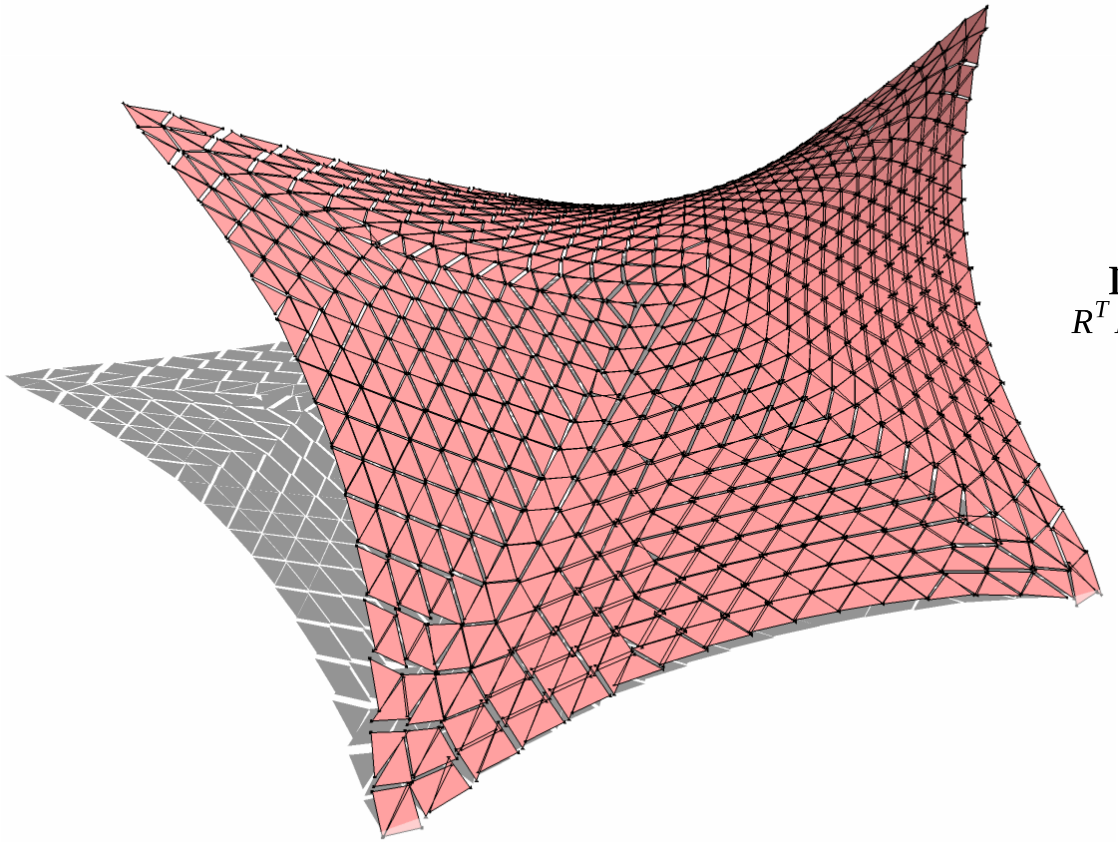
Nonlinear Optimization

Clustering



$$\min_{C_j, ind} \sum_i D(P_i, C_{ind(i)})$$

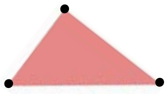
Clustering



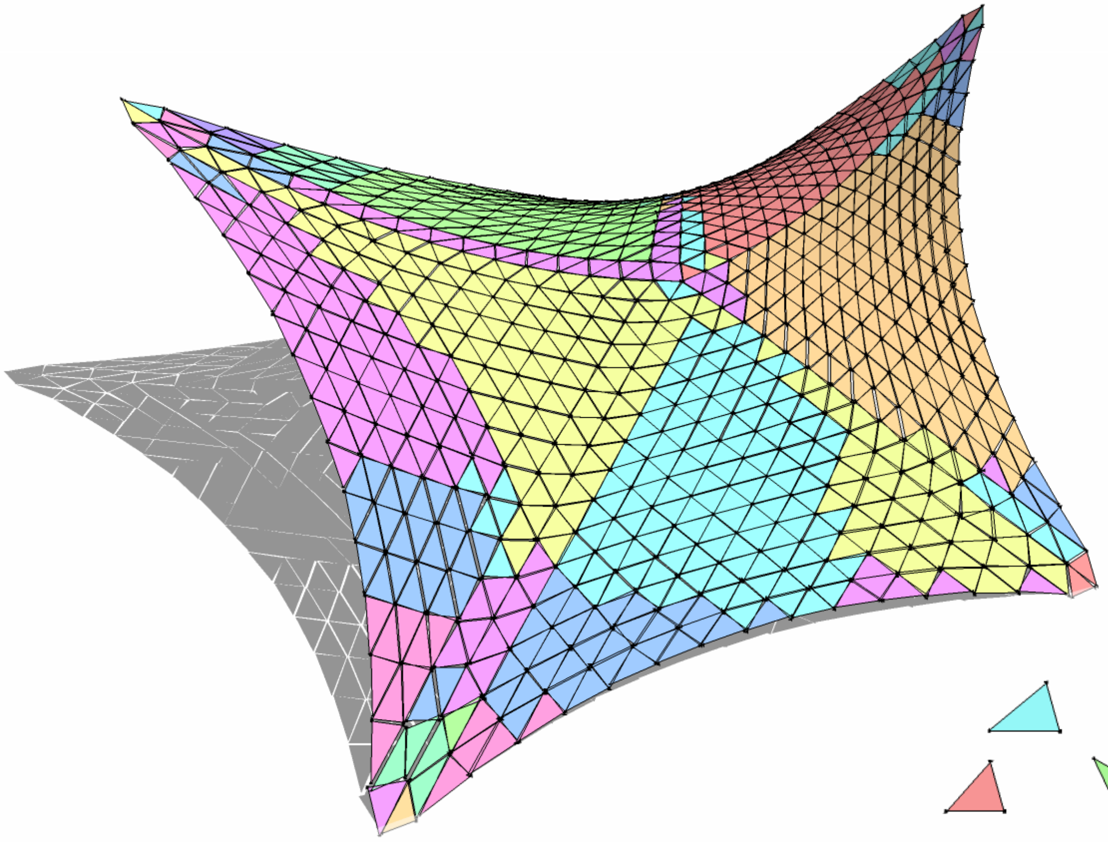
$$\min_{R^T R=I,T,j} \sum_{l=1}^{3} | R\boxed{C}_{perm(j,l)} + T - \boxed{P_l} |^2$$

Rigid Transformation

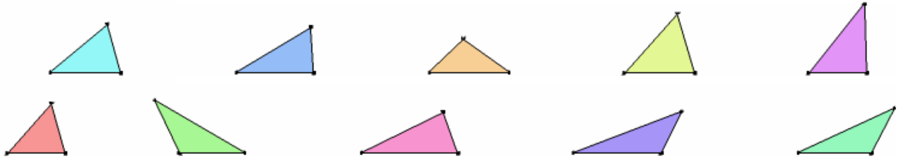
1280 triangles | **1 cluster**



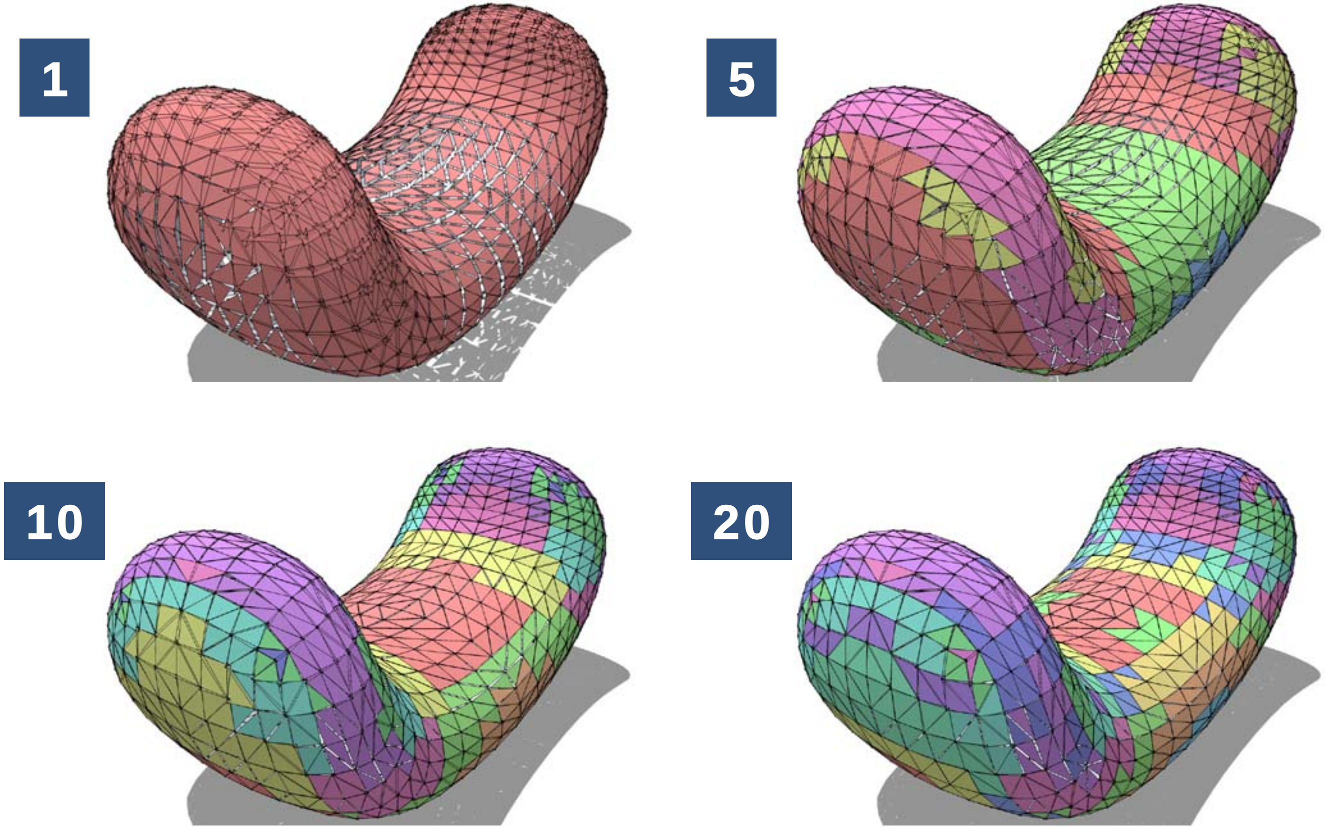
Clustering



1280 triangles | **10 clusters**



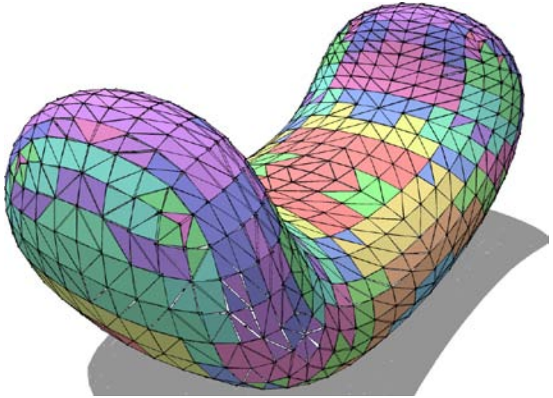
Varying the Number of Clusters



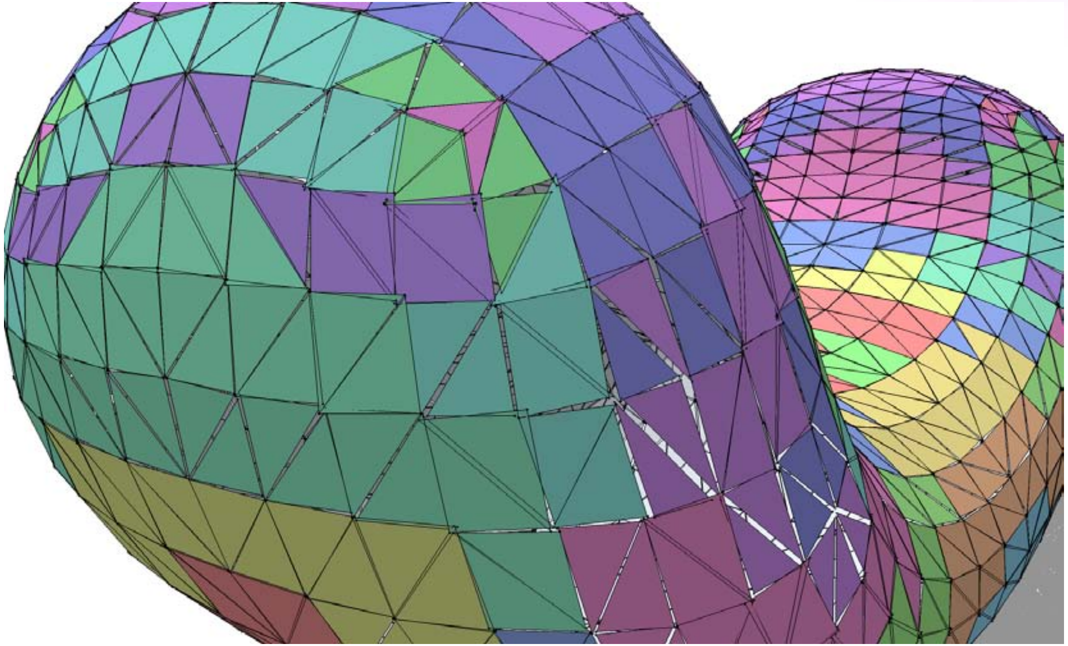
Before Global Optimization

Spacing between Triangles

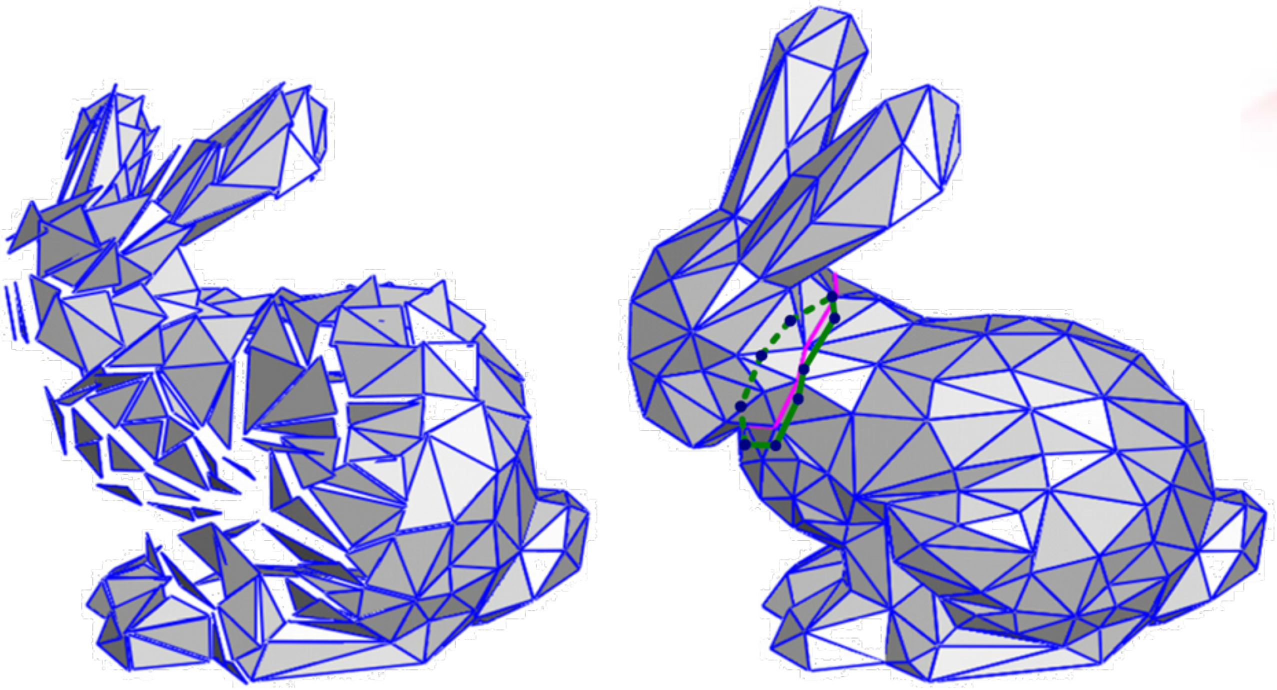
20 clusters



Before Global Optimization

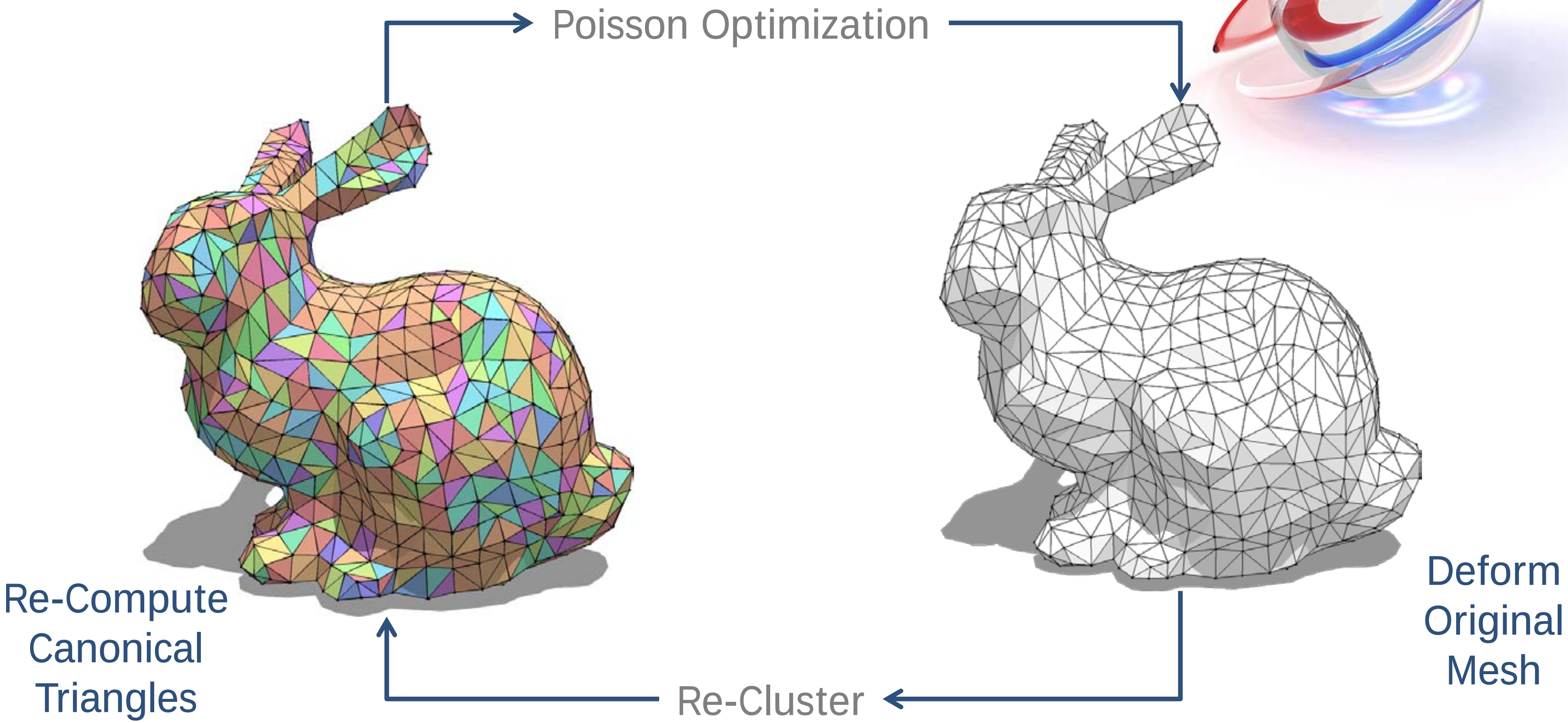


Disconnected Triangles



Poisson Optimization - Yu et al. [2004]

Global Optimization



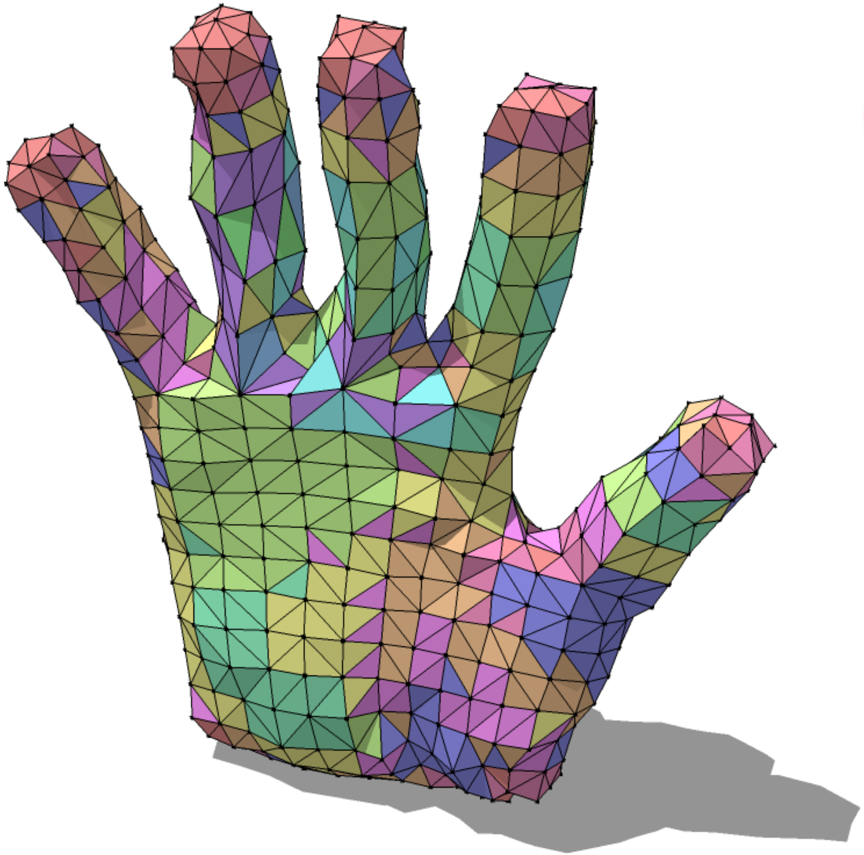
Global Optimization

$$\min_{P}(E_g + \alpha E_c + \beta E_b)$$

Gradient

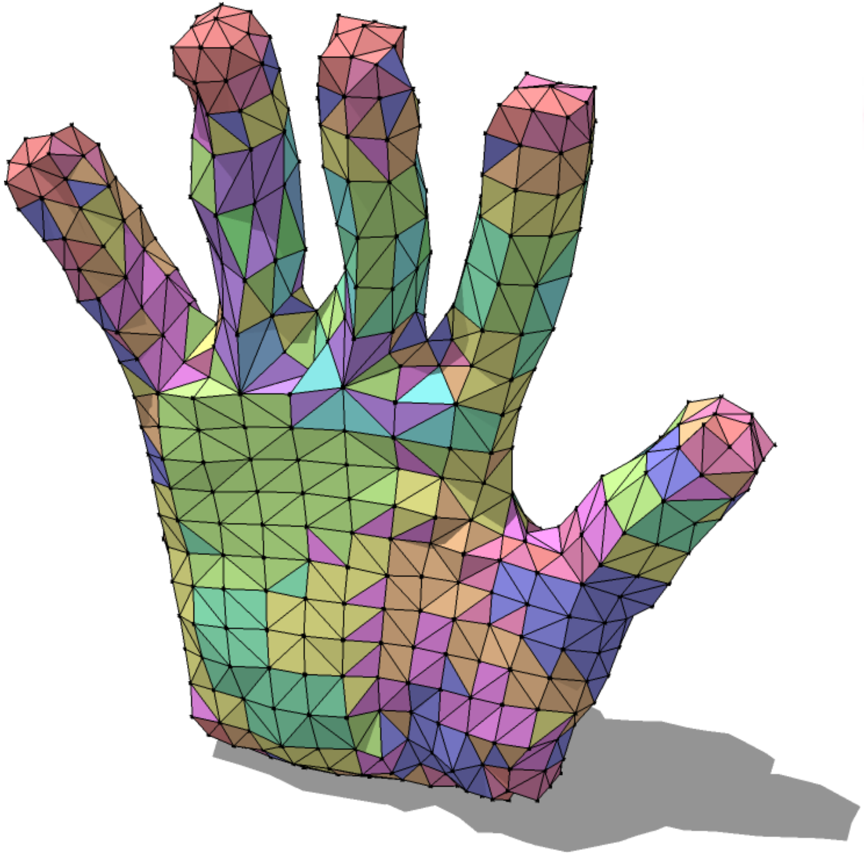
Proximity to original shape

Proximity and Fairness



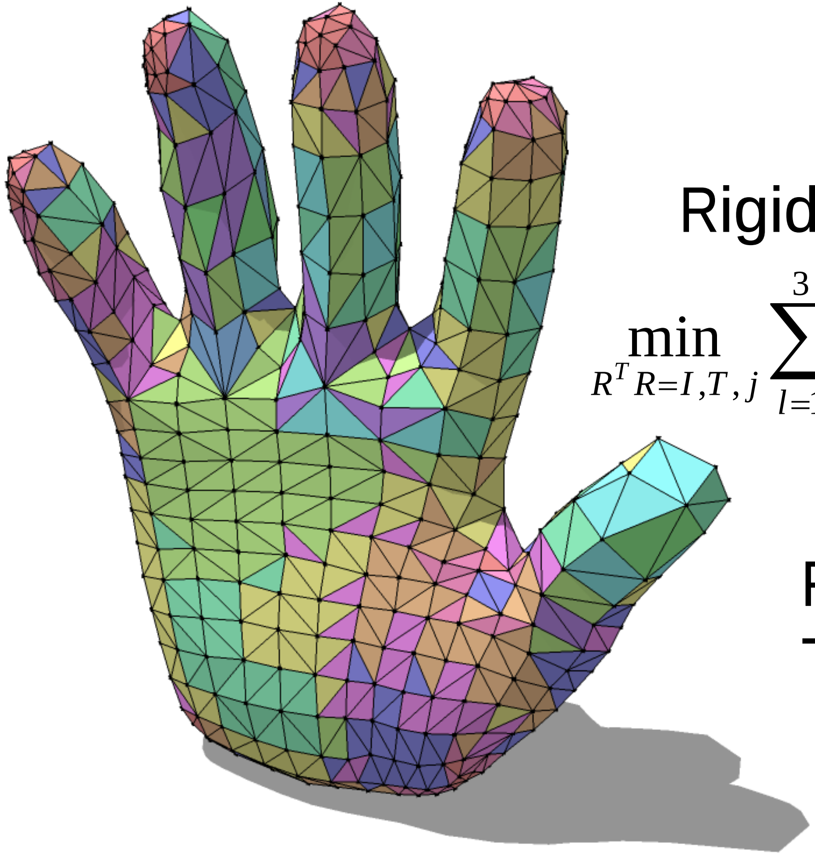
Proximity and Fairness

Global
Non-Linear
Optimization



Proximity and Fairness

Global
~~Non-~~Linear
Optimization



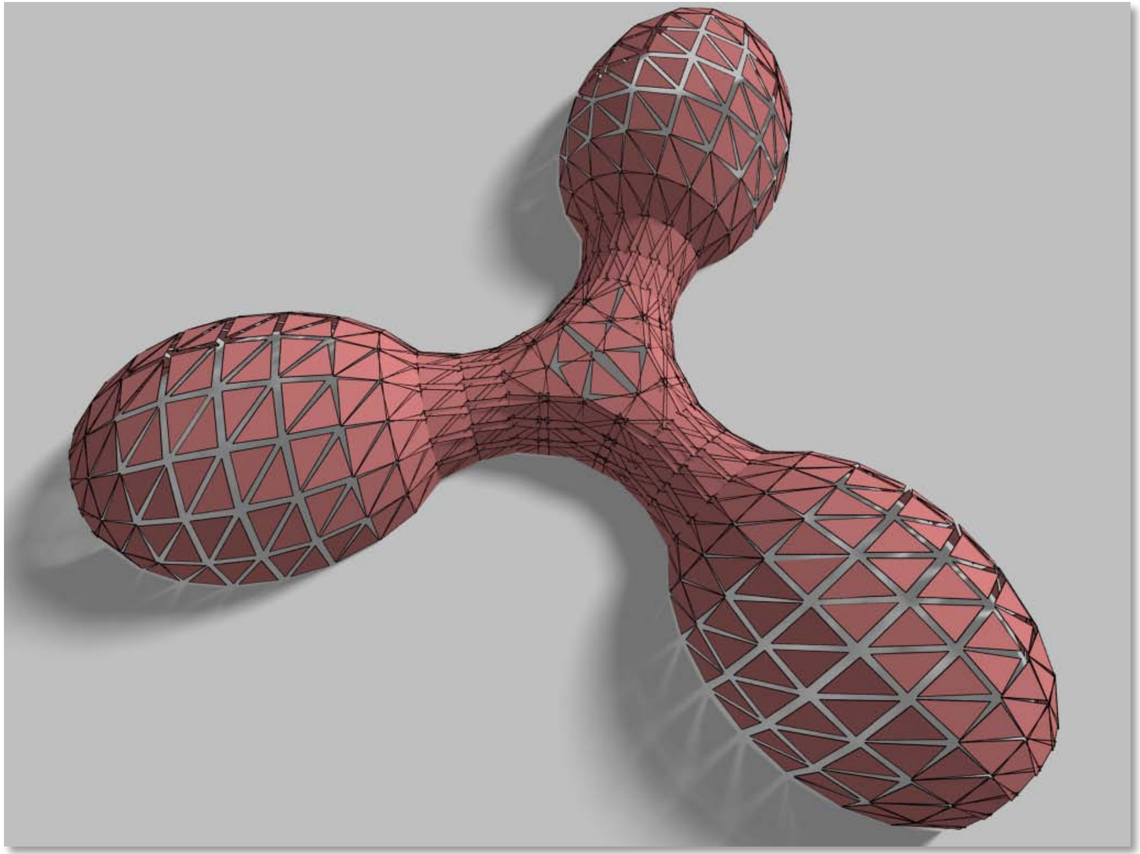
Rigid Transformation

$$\min_{R^T R=I,T,j} \sum_{l=1}^{3} | R\boxed{C}_{perm(j,l)} + T - \boxed{P_l} |^2$$

+

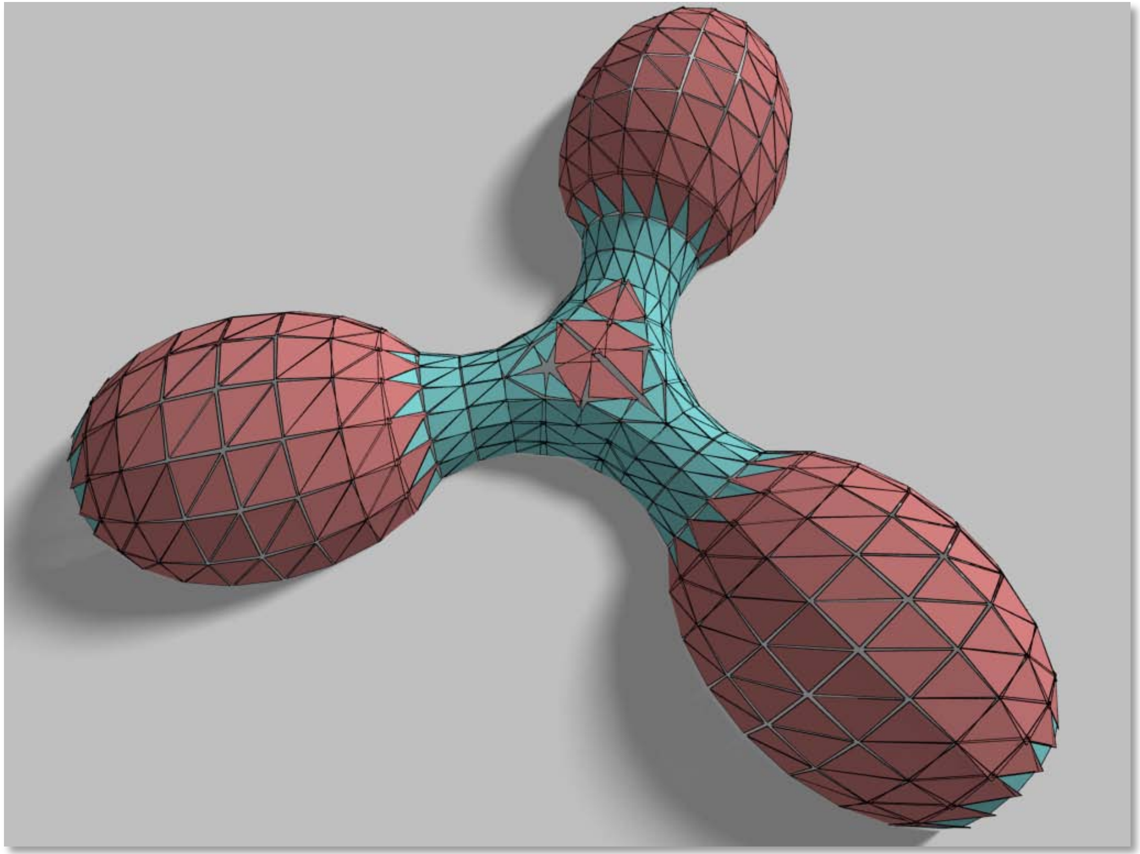
Rotate Canonical Triangle

1 - Cluster

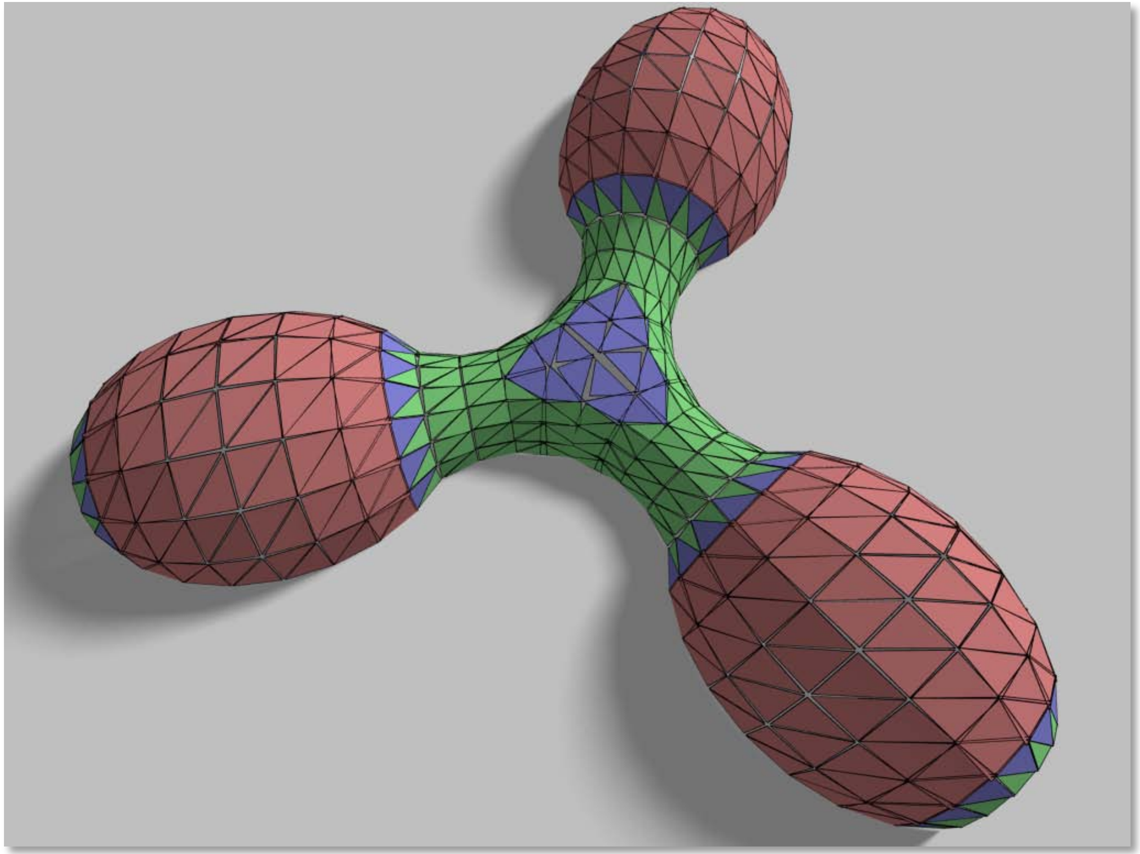


Architectural Dome
576 Triangles

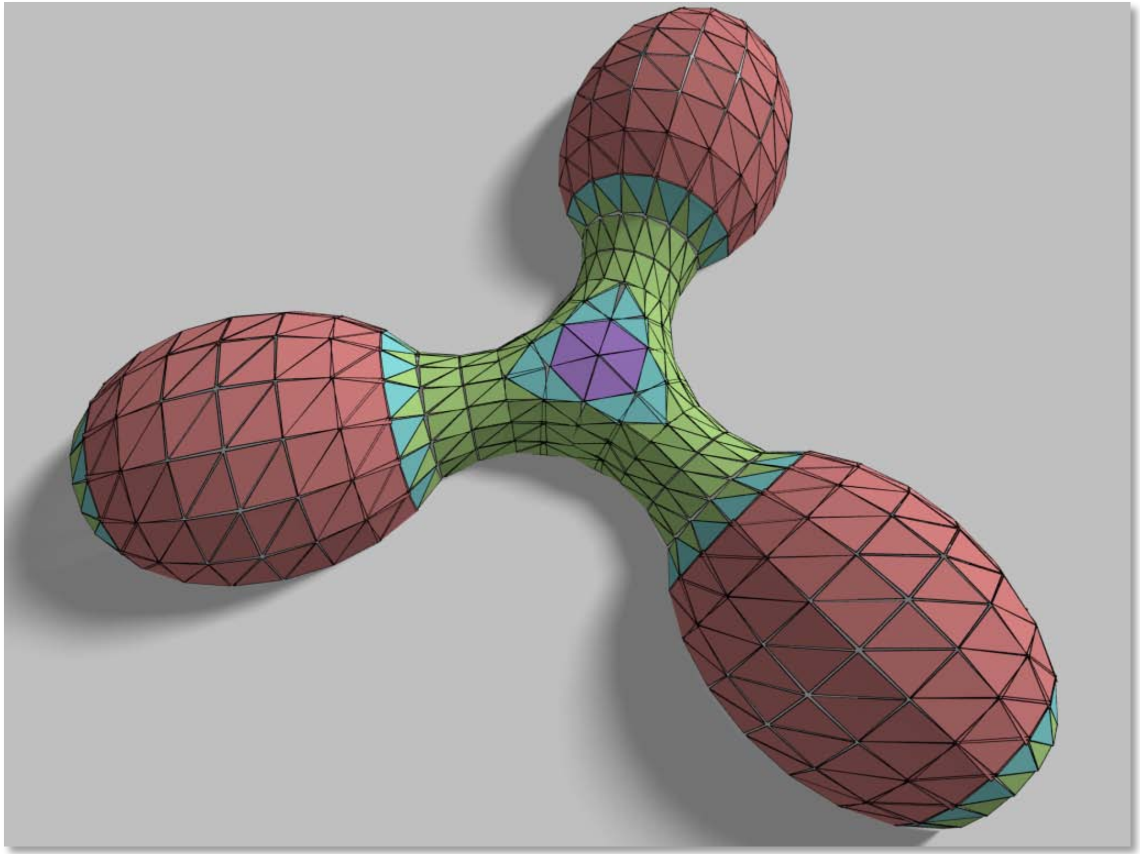
2 - Clusters



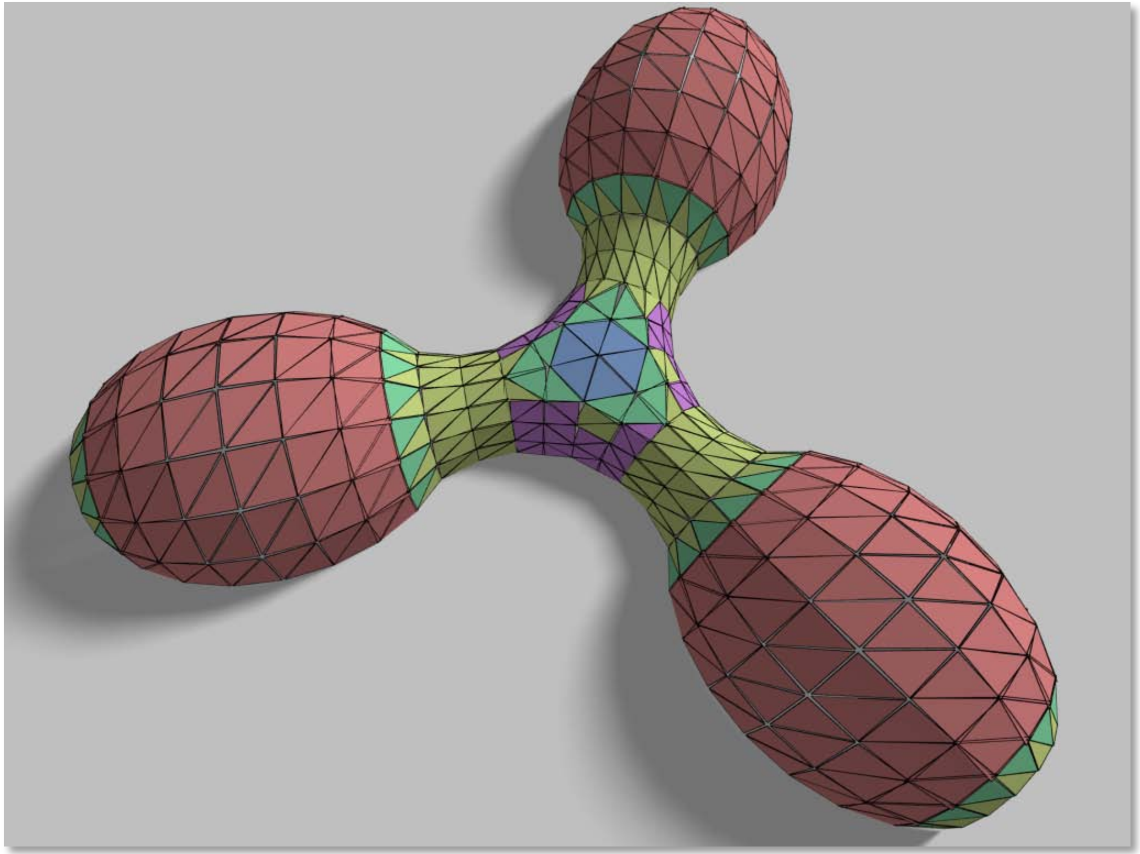
3 - Clusters



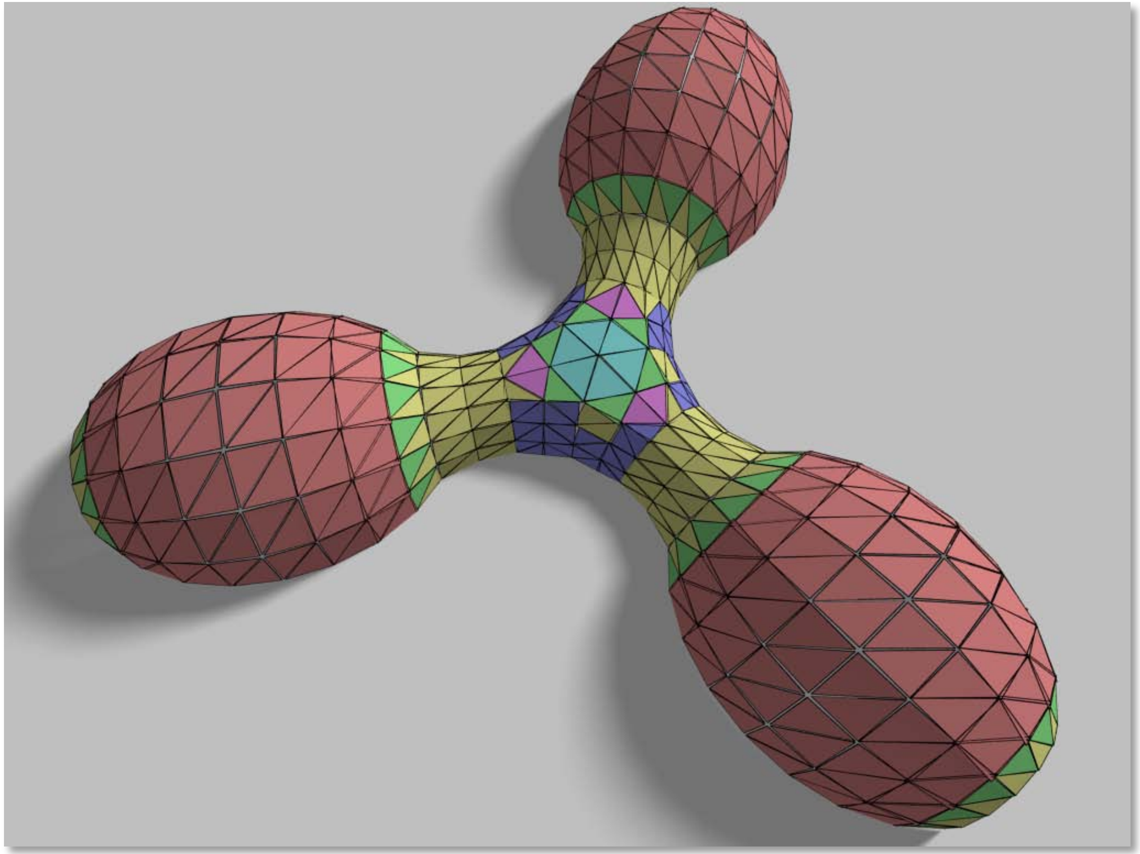
4 - Clusters



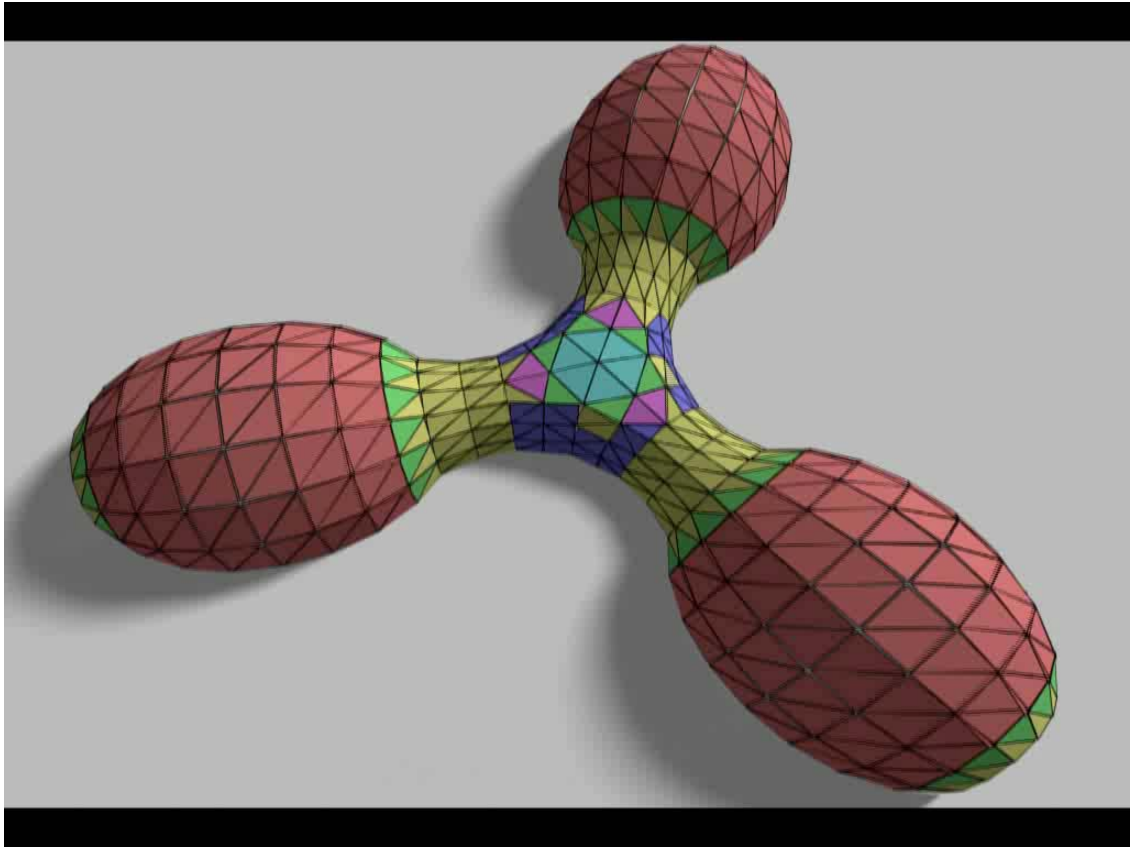
5 - Clusters



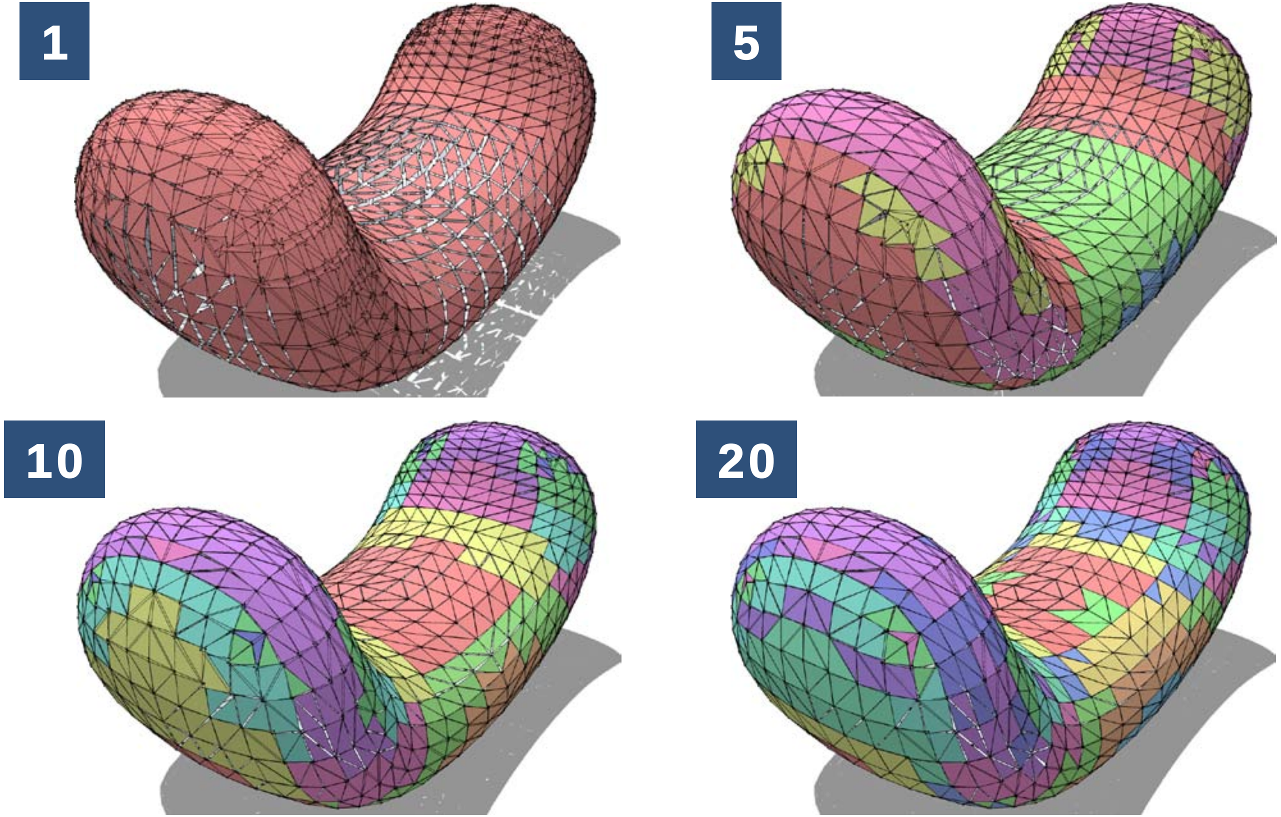
6 - Clusters



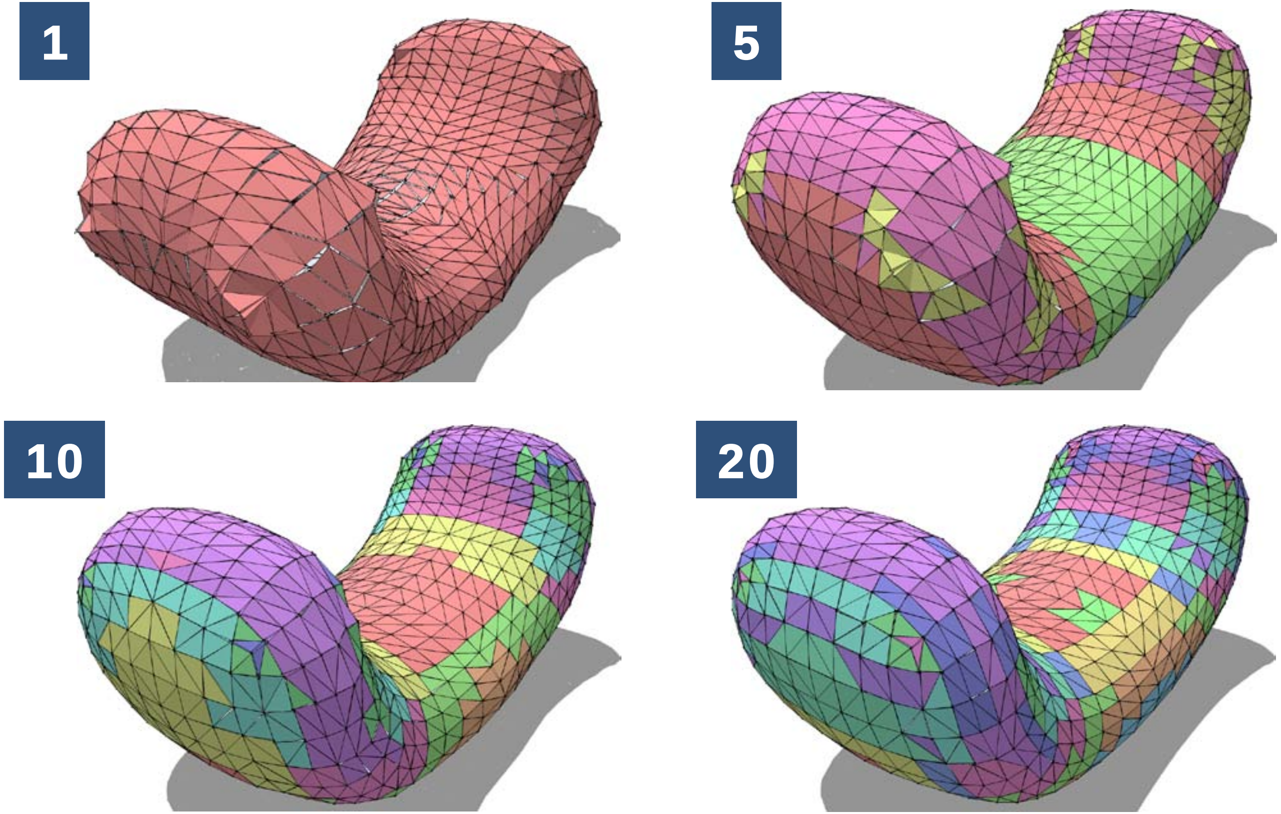
Clustering & Global Optimization



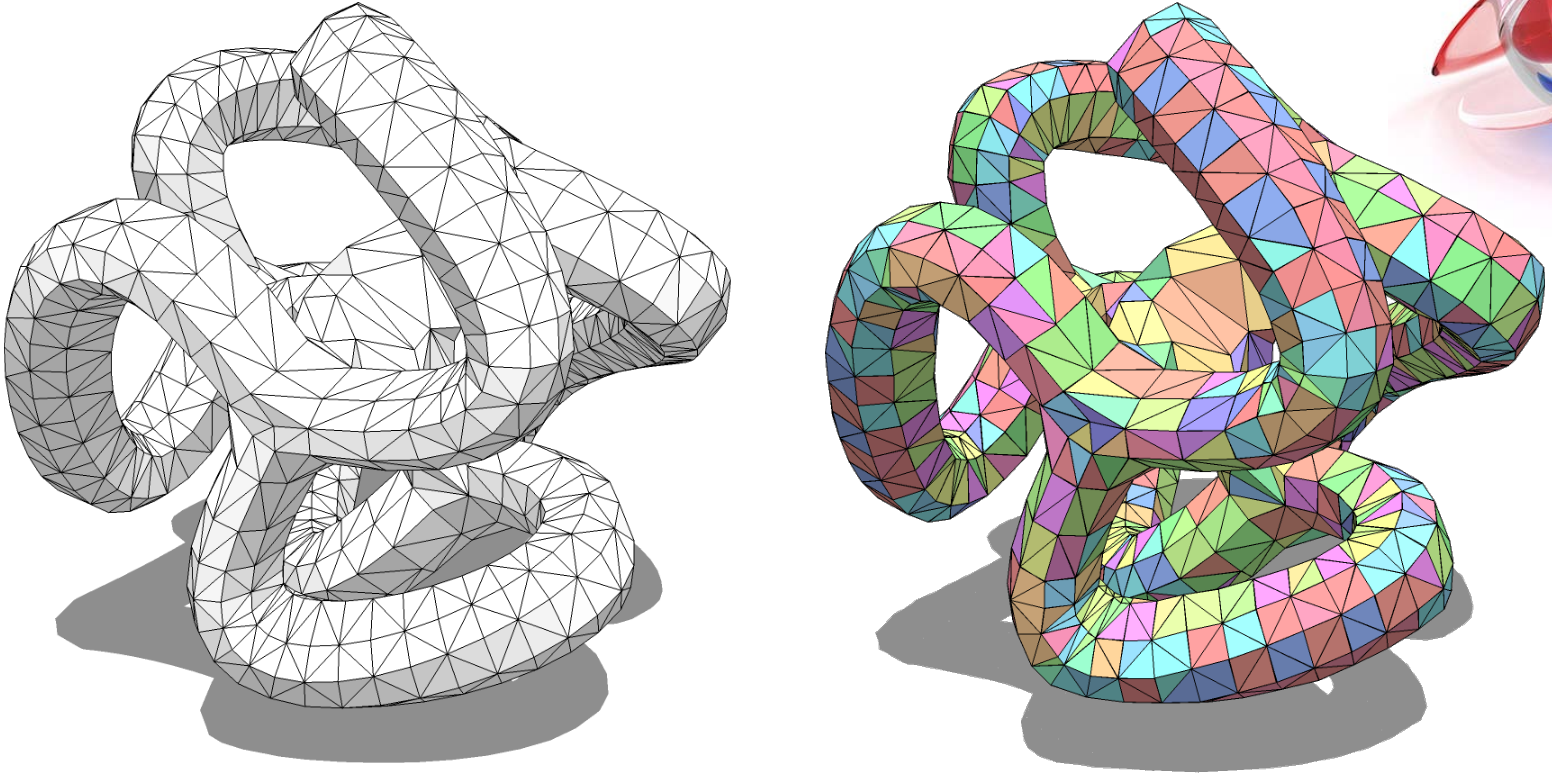
Before Global Optimization



After Global Optimization



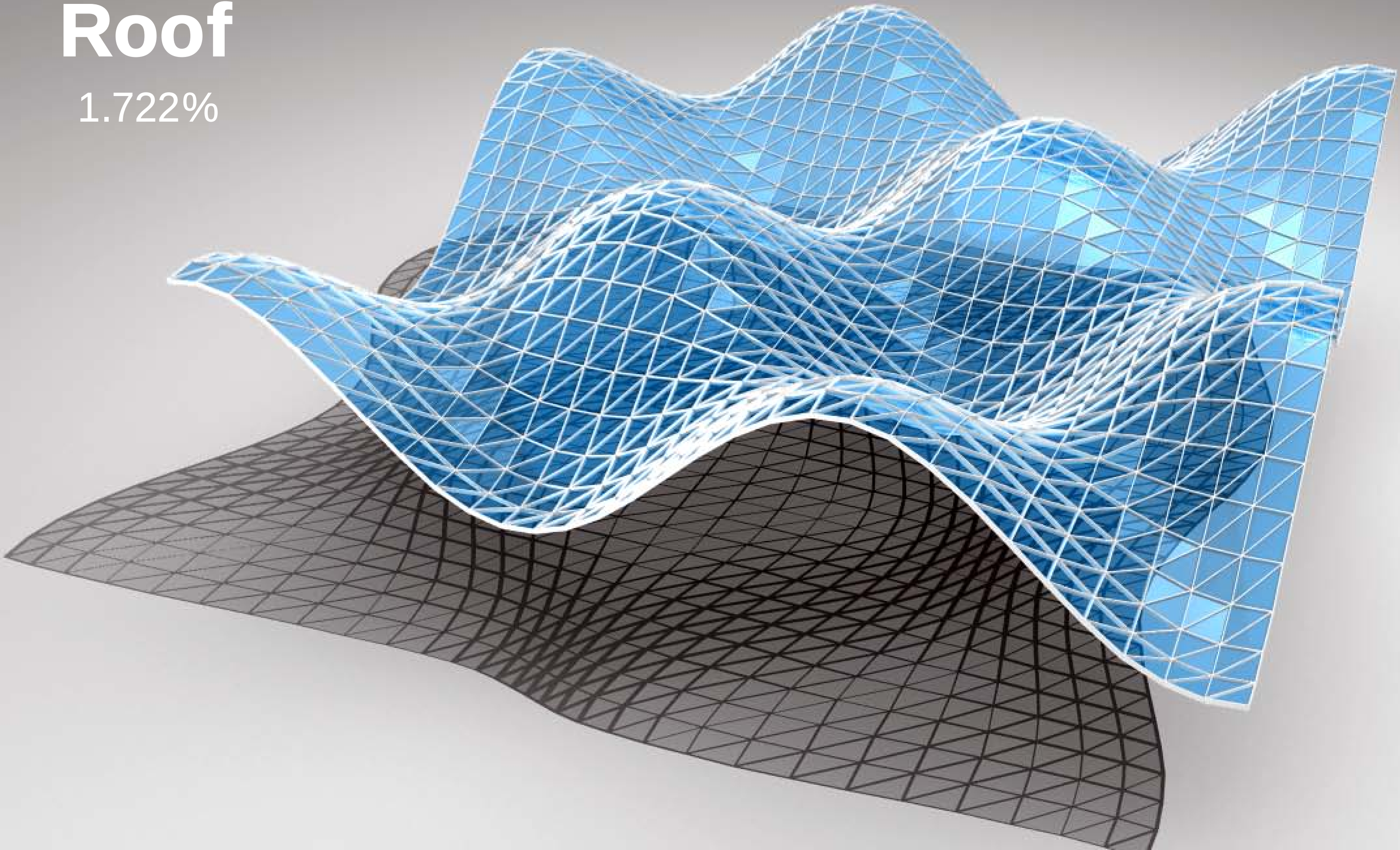
Example



2492 triangles | 64 clusters = 2.56% of total triangles

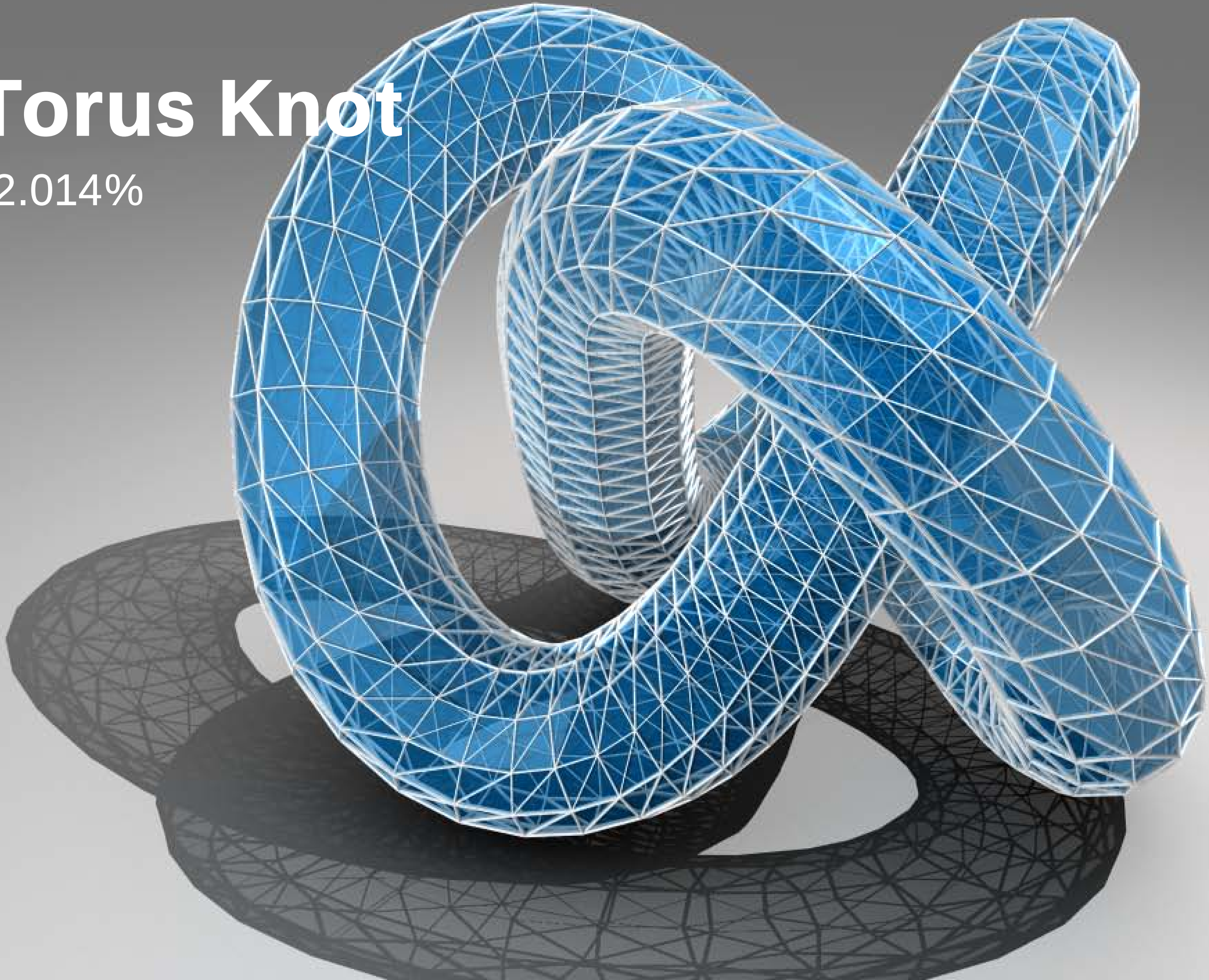
Roof

1.722%



Torus Knot

2.014%

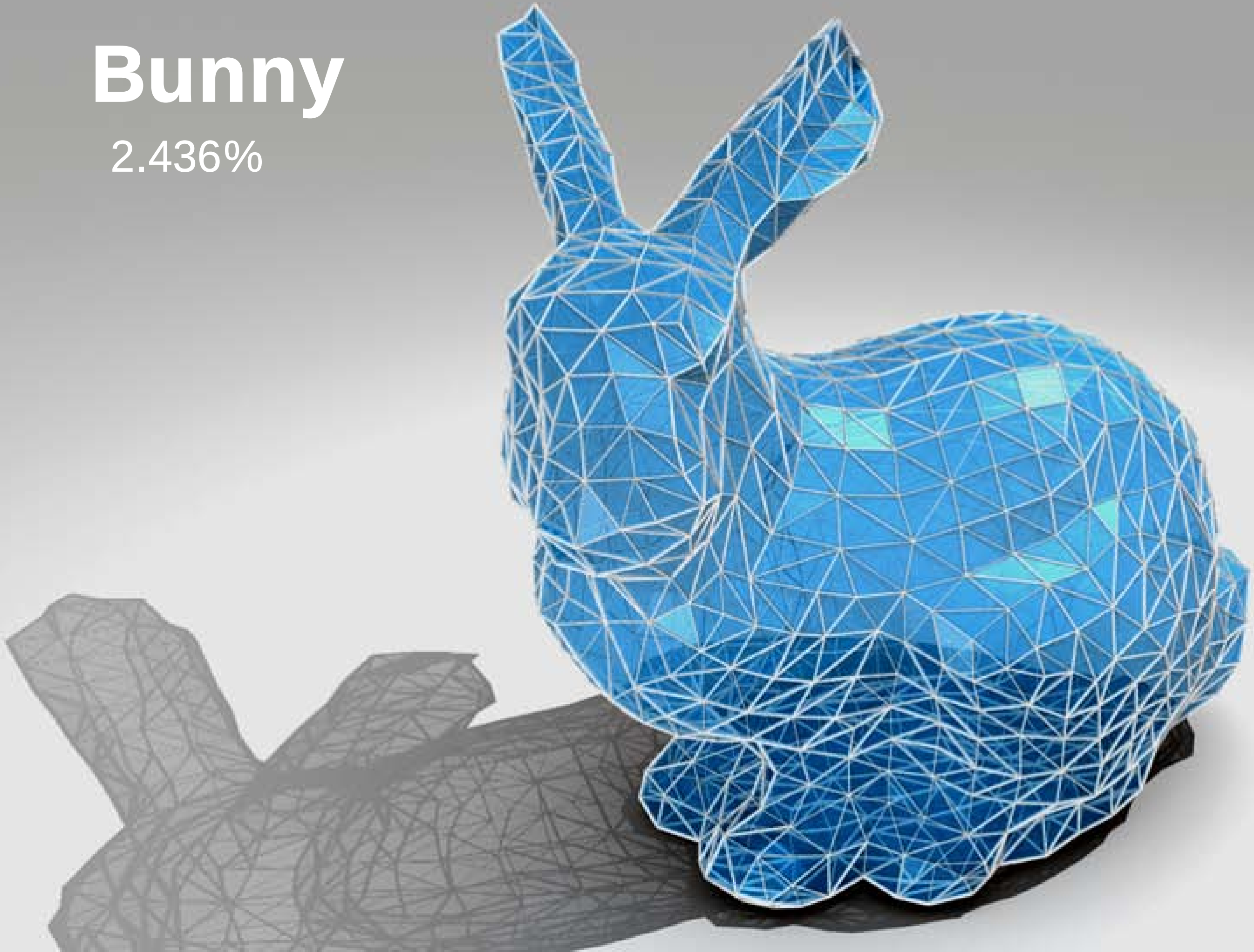


Venus

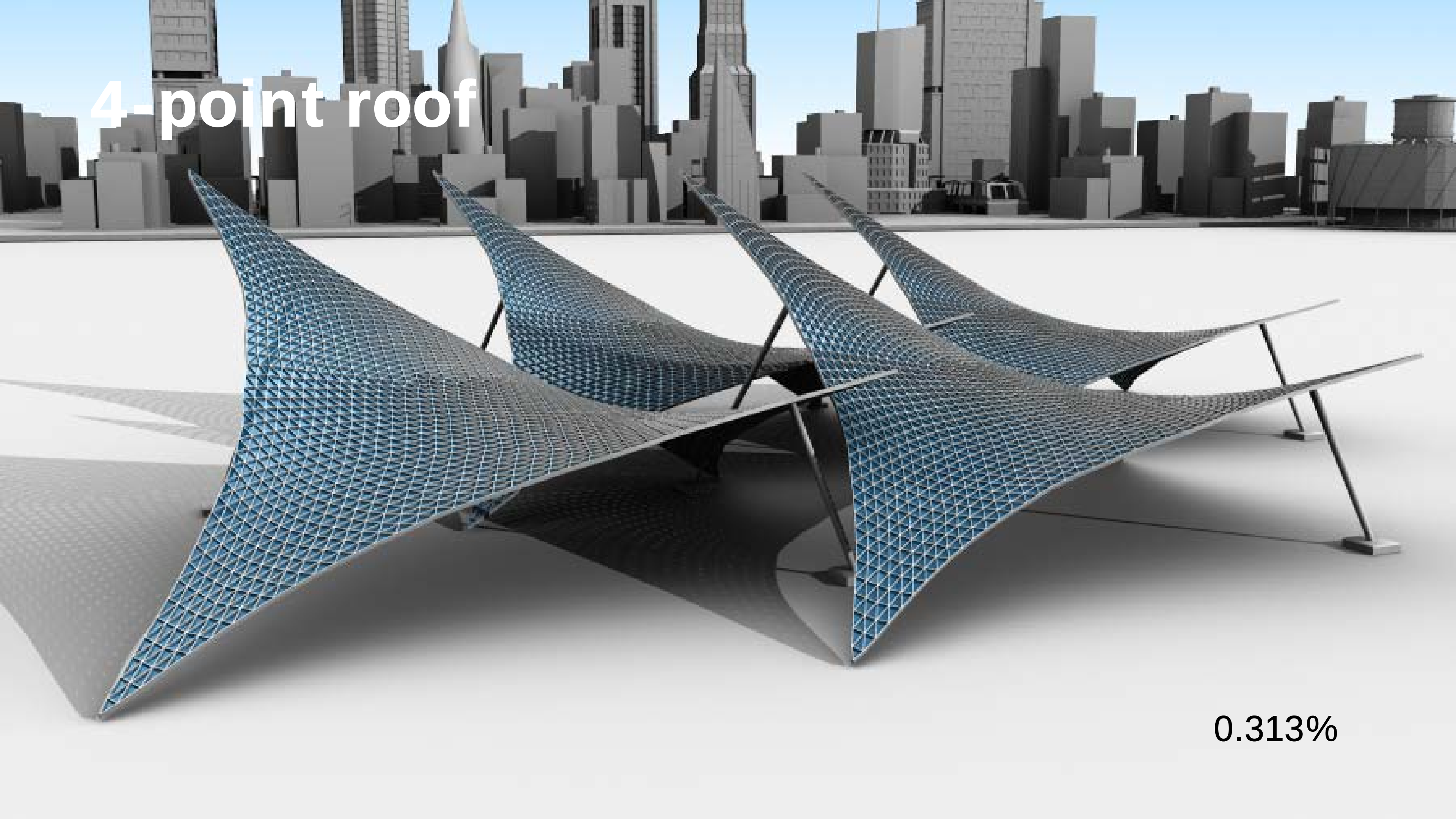
6.017%

Bunny

2.436%



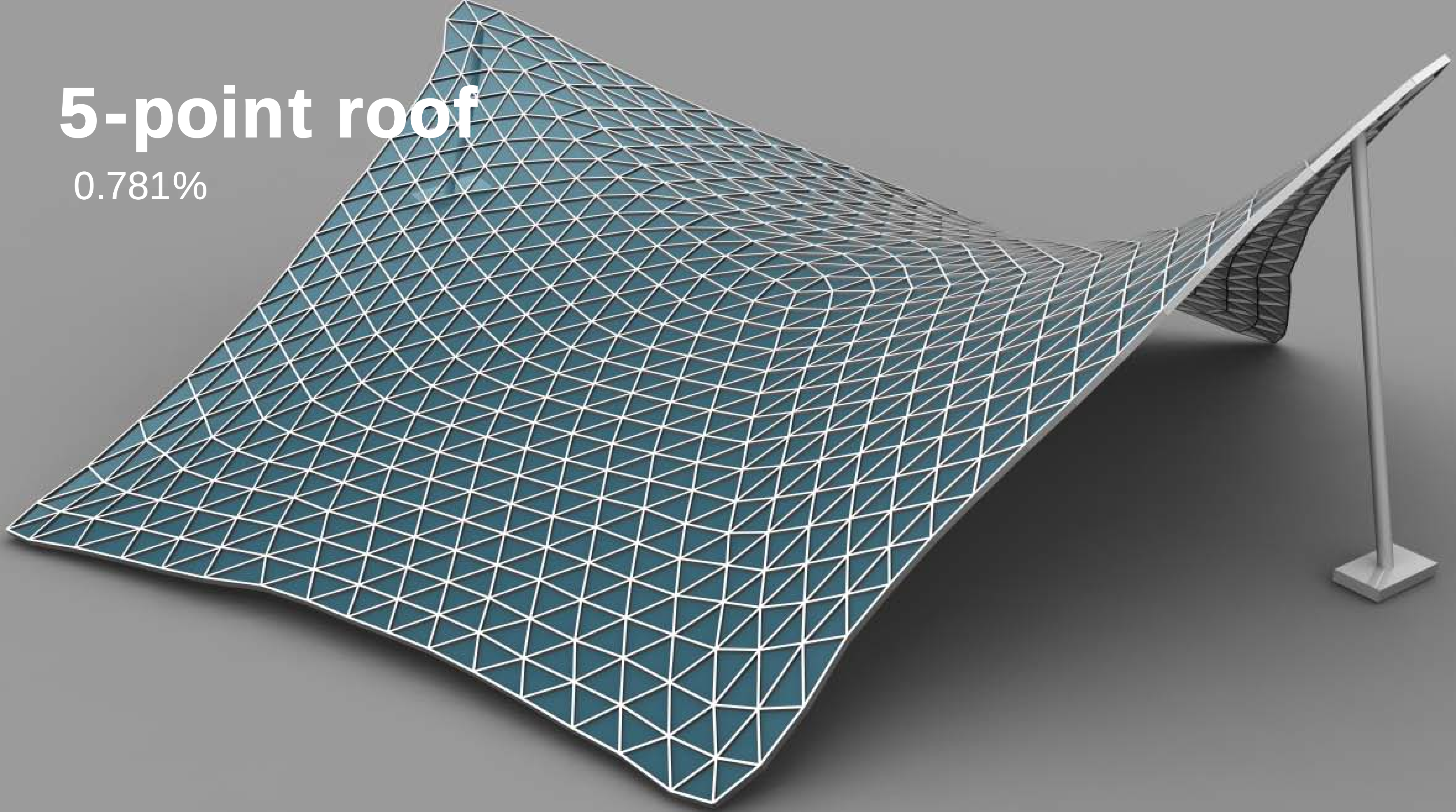
4-point roof



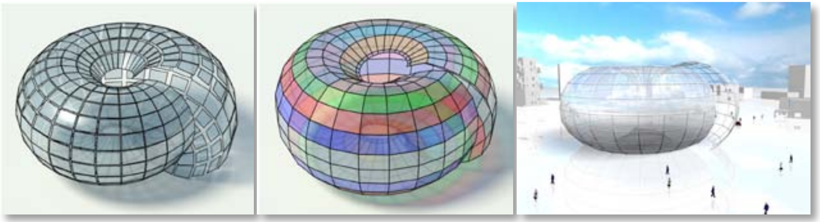
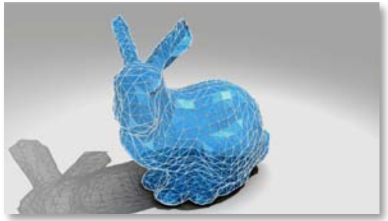
0.313%

5-point roof

0.781%

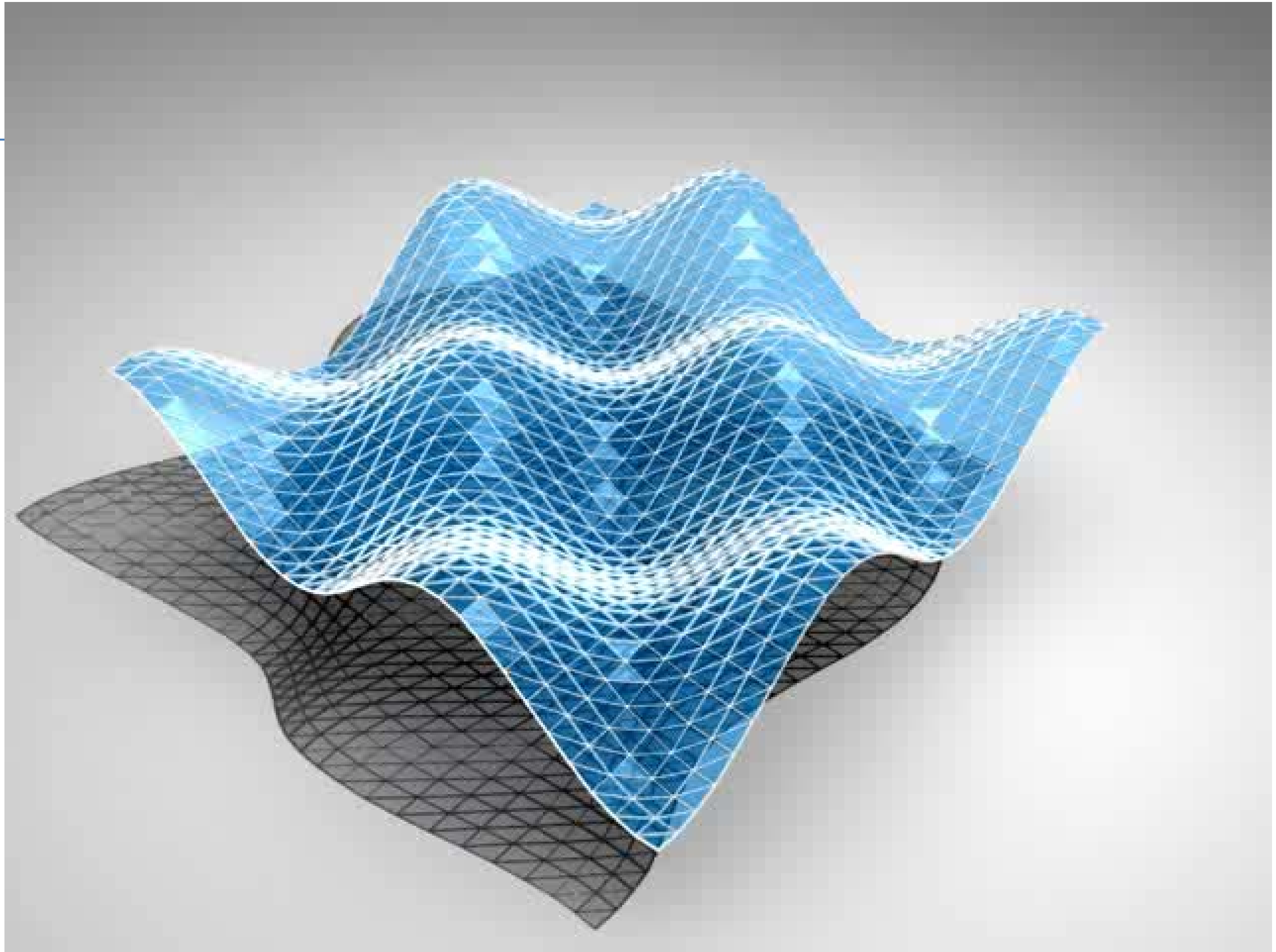


Comparison

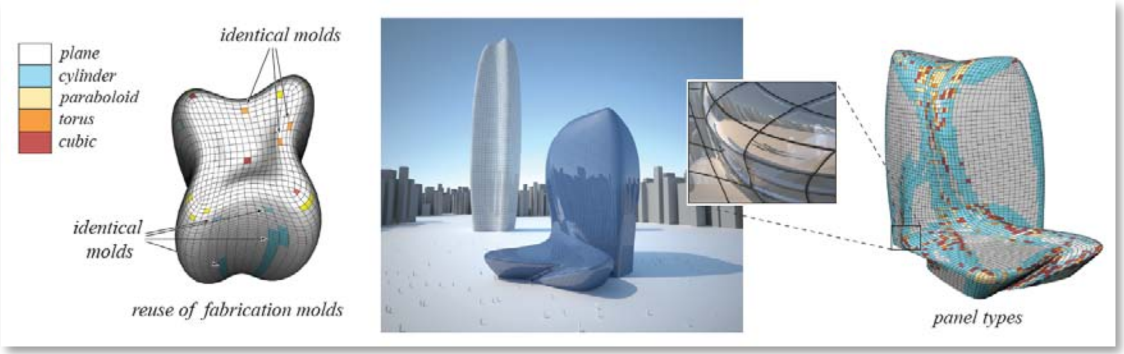
K-set Tilable Surfaces	Ours
	
Non planar Quadrilaterals	Planar Triangle
8 permutations for best rigid transformation	6 permutations for best rigid transformation
Mean S-quad, compute once	Non linear search for canonical, iterative
Global non-linear optimization	Global linear optimization
Begin with large # of clusters & merge	Begin with small # of clusters & add more

Future Work

- Detect outliers in clusters
- n-gons
  - Planarity
- Modify topology
  - Symmetry?
- Maintain streamlines
  - Non-existent?

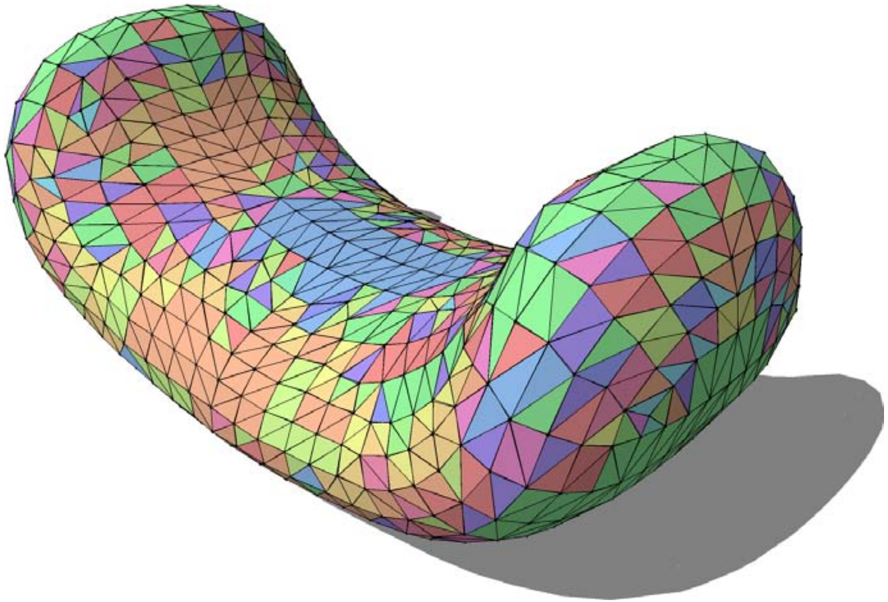


Paneling Arch. Freeform Surfaces

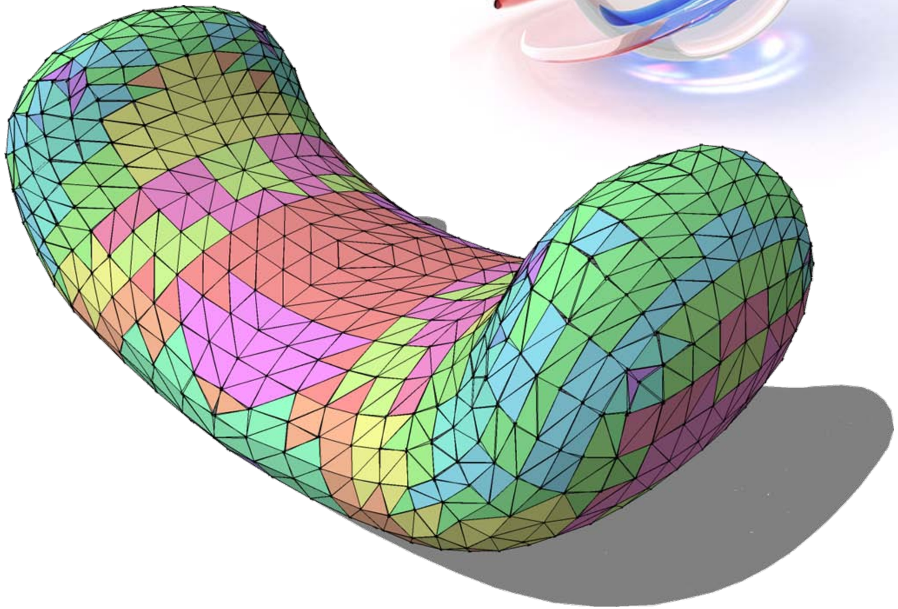


- Use small # of molds, with associated cost
- Create non-congruent panels from the mold
- Emphasis upon streamlines
- Minimize divergence and kink angle

Clustering

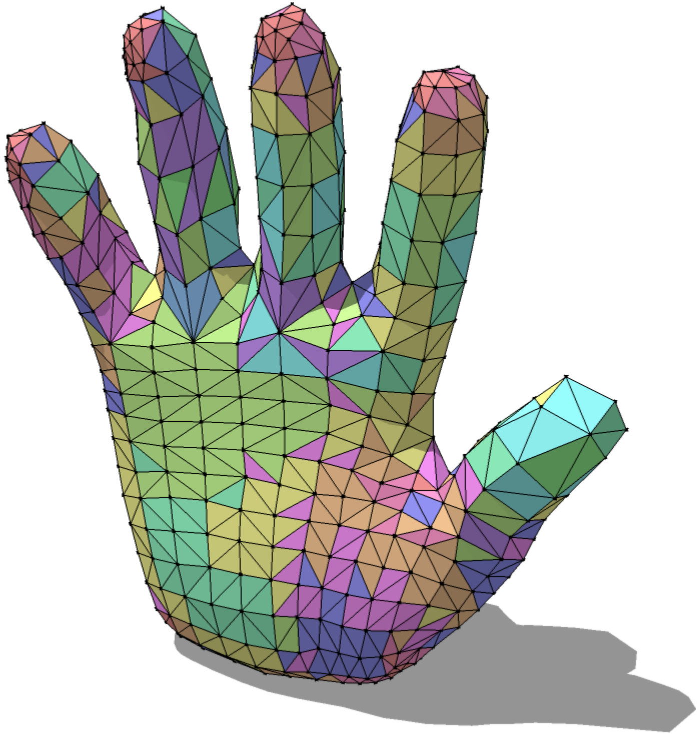


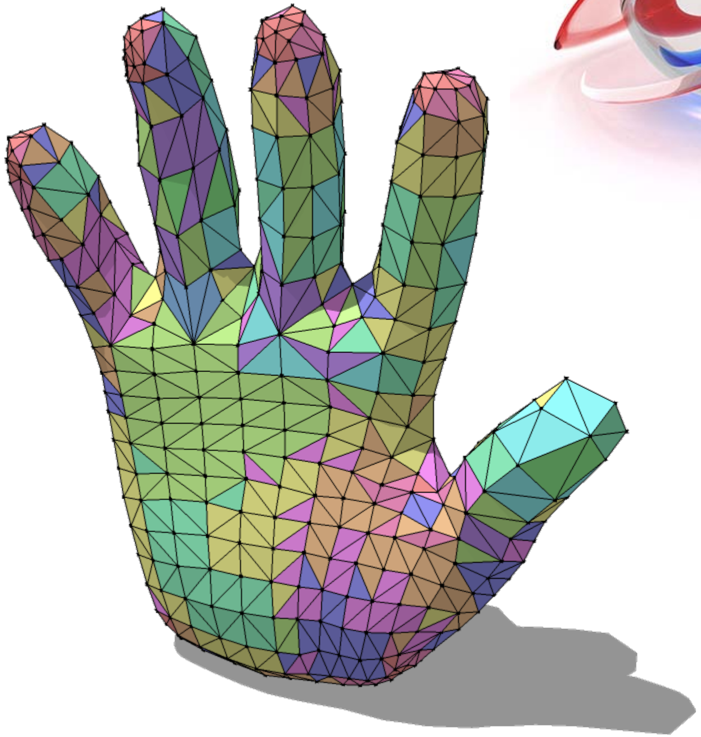
Adding 1 Cluster incrementally and running optimization to convergence



17 Clusters before running global optimization to convergence

Rotation of Canonical Triangle


50% rotation


100% rotation

Comparative Analysis

Paneling Architectural Freeform Surfaces	K-set Tilable Surfaces	Triangle Surfaces with Discrete Equivalence Classes
• Use of small # of molds • Each mold has an associated cost • Emphasis upon streamlines • Divergence and Kink angle	• Non-planar quads • 8 permutations for rigid transformation • Global non-linear optimization • Start with large # of clusters and merge • Mean S-quad, computed once	• Planar Triangles • 6 permutations for rigid transformation • Global linear optimization • Begin with 1 cluster, add more • Non linear search for canonical triangles, updated for each iteration