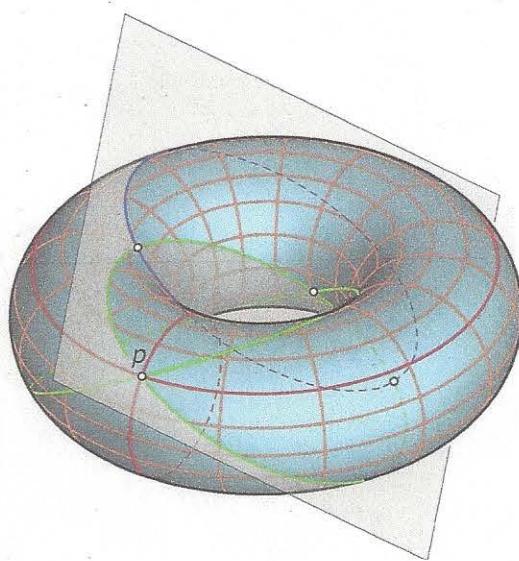


ng torus.  
a torus.  
intersecting

\*Example:

**Villarceau's circles of a torus.** Among the three torus types the ring torus has a remarkable property. In addition to the meridian and the parallel circles, a ring torus carries two other families of circles. Each *double tangent plane* (i.e., a plane tangent to the torus at exactly two different points) intersects the surface along two circles. They are named *Villarceau's circles* in honor of their discoverer the French mathematician and engineer Yvon Villarceau (1813–1883). Thus, every point  $p$  on a ring torus can have four circles drawn through it (Figure 9.10).

or a  
doubly  
circle?!



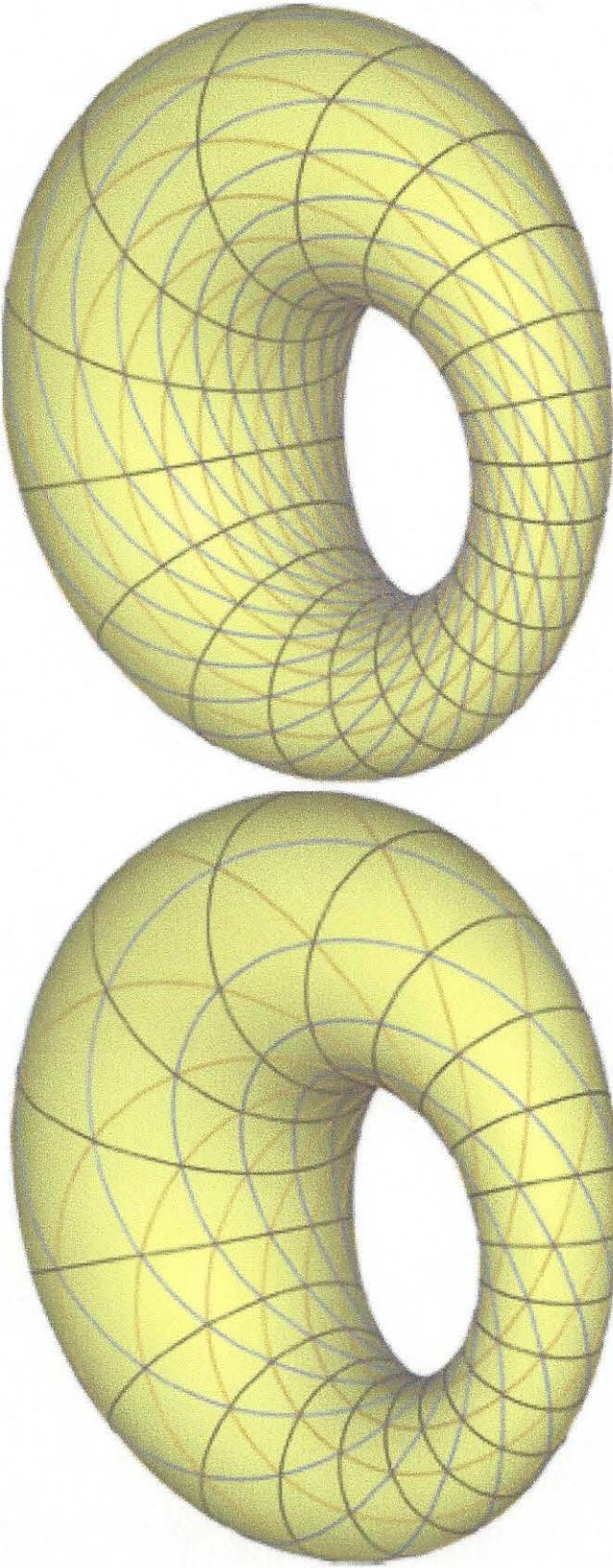


Fig. 16. Since the Villarceau circles on a Dupin ring cyclide intersect the principal curvature circles under a constant angle, there are several ways of extracting circle families with a constant intersection angle. This leads to congruent nodes in an architectural construction. We show here two different types of 3-webs with congruent nodes, as discussed in Theorem 17.

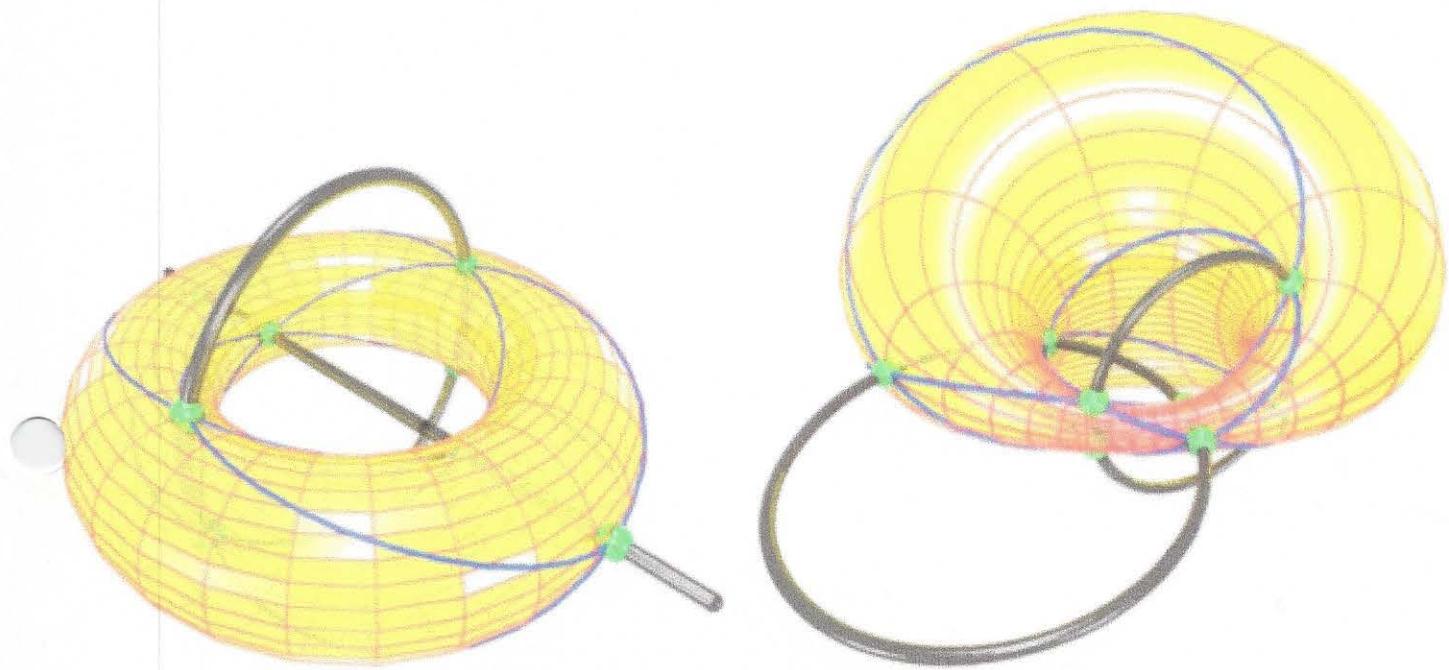


Figure 11: Left: Ring torus with a Villarceau circle and points joined by a 4-ortho-circle (Lemma 5.4). Right: Ring shaped Dupin cyclide with 4-ortho-circles joining points on a Villarceau circle (Lemma 6.8)